THE USV ANNALS OF ECONOMICS AND PUBLIC ADMINISTRATION | VOLUME 13, ISSUE 2(18), 2013

AN ADVERTISING OLIGOPOLY

Ph.D. Student **Alina Irina GHIRVU**
Faculty of Economical Sciences and Business Administration Babeş-Bolyai University, Cluj – Napoca, Romania
alina.ghirvu@ubbcluj.ro

Abstract:
This paper proposes a model of advertising competition based on the Cournot oligopoly model using a dynamic system, the equilibrium points of which can be determined analytically. We consider several cases for explaining the way in which firms will adapt their own advertising volume, depending on the number and the advertising volume of their competitors, in the context of an online video game used for advertising purposes. The dynamic setup is given by online Internet connection that allows interaction and communication and the high level of technology that permits nowadays real-time advertising insertions.

Key words: advergame, dynamic system, Cournot oligopoly, advertising, video game

JEL classification: C72

1. INTRODUCTION

The development of computer and Internet technology made video games become highly complex and interactive. Companies interested in new marketing extensions from classic models gained upon their competitors by using video games as a communication tool for increasing receptiveness of the brand message.

Video games were first studied in the academic world only recently, the year 2001 being considered to be the moment when video games began to be treated “seriously”. The fast impact on the audience made video game become a real breakthrough in advertising and a powerful competitor for traditional media as television and movies.

Advertising and video games meet in two major ways: advergames and in-game advertising. An advergame represents a video game constructed around a brand or product, sponsored by a marketer and constructed for delivering the brand messages. It has an immersive mix of advertising and entertainment and can be accessed online through corporate or brand websites.

In-game advertising represents an integration of advertising messages in a video game that has a story already created and is coordinated and sold by independent gaming organizations, using dynamic ad-insertion networks (Winkler and Buckner, 2006).

The purpose of advertising using video games is to increase brand awareness and familiarity with the brands and products advertised, to build long time relations with the consumers, positive feelings, sympathy and fidelity and to increase purchase intention that will be later transform in actual sales and increasing profits.

2. BACKGROUND

The existing literature regarding dynamic models of advertising includes multiple studies on how advertising works. Studies about direct relations established between the sales rates of change and the advertising effort of firms have their origins back to Vidale and Wolfe (1957). Most relevant works approach the issue of optimal advertising in a duopolistic market discussed by Leitmann and Schmitendorf (1978).

There were several attempts to bring dynamic analysis of advertising competition to an oligopoly setting, in which more than two competitors are involved. The model that Feichtinger (1983) proposed was an extension from a differential game between two profit maximizing firms to the case of general effectiveness functions for advertising. Analysis of Nash equilibrium solutions was applied to the Leitmann-Schmitendorf advertising model for investigating the structure of optimal advertising rates. By setting up a dynamic advertising model in a duopoly competition setup, Erickson (1985) provided important aspects on the ways that advertising can operate in competitive and dynamic markets.

A group of studies about dynamic models of advertising originates from Nerlove and Arrow (1962) and deals with using advertising for increasing the stock of goodwill and recommendation, considering that the cumulate effect of past and present advertising expenditures influences the current demand for the goods that the firm provides.
His model was later used in studies of Sethi (1977), Fersthman (1984) and Jorjensen (1999).

The game theory groundwork of dynamic differential games allows to forecast strategy adopted by the players and to anticipate their moves. Studies about advertising strategies used differential games to analyze current outcomes of a firm, advertising decision effects and advertising implications in the future of a brand or product. Many differential game models applied numerical, analytical, qualitative and empirical methods for studying situations that involve dynamic advertising competitions (Erickson, 1995).

The problem of optimum in advertising was treated in terms of promotional budget allocation in a two-person zero-sum game with a mini-max solution and pure strategy game (Friedman, 1957). The author considers that the major factors governing advertising allocation are competitive expenditures and that the competitive effect is a key element for determining real advertising budget allocations.

Advertising dynamic games ware discussed also in a two-person, nonzero-sum stochastic games to analyse dynamic advertising models with a discrete-time Markov decision process (Dirven and Vrieze, 1985)

Researchers investigating advertising competition models used the Cournot oligopoly model as a way of analyzing different matters involved in the decision making process. Differential oligopoly games were discussed with different setups: when firms which are engaged in a Cournot competition on a market with homogeneous goods and invest in advertising for increasing the consumer's reservation price (Cellini and Lambertini, 2003). Authors continued this research with a model of differential game of advertising under both Cournot and Bertrand competition (Cellini et. all, 2008).

Simulation models based on the Cournot analytical model of duopoly were used in investigating the field of industrial organization economics (Martin, 2002). Several studies (Dixit, 1979; Perloff, 2008) determined the Cournot equilibrium using an analytical approach.

In a study on how benefits of mandated generic advertising vary with firm size in an asymmetric Cournot oligopoly market, with a focus on unit-assessment funding, researchers found that "the effect of such a program on an individual firm's profit depends on the nature of the change in market demand, and also on the firm's market share" (Zheng et. all, 2009).

As can be seen, previous studies regarding the effect that video games have on consumer did not approach game theory models for investigating the way that advergames or in-game advertising works. Furthermore, there are no previous studies in the field that are using a Cournot oligopoly model for explaining the dynamic process of advertising volume adjustment for firms that advertise through video games.

3. GENERAL CONSIDERATIONS ABOUT THE MODEL

In this paper we try to set up a model based on the Cournot oligopoly model for explaining the way in which firms will adapt their own advertising volume, depending on the number and the advertising volume of their competitors.

We consider several cases that can be investigated in a video game context. The dynamic setup is given by the internet connection that allows real-time interaction and communication and the high level of technology that is used nowadays for controlling and ordering online insertions.

Further, we investigate only the advertising effect on sales, so the price will be neglected. We assume that any extra advertising unit will create supplementary effect on consumers and that firms will insert as much advertising units that the environment allows.

After a certain advertising exposure level, the player's memory will become agglomerated and it will generate resistance. In other words, a higher volume of advertising will immunize the consumer. This can happen under excess exposure of a single or multiple brands.

During constant gaming sessions, players are constantly exposed to advertisement insertions, which can take the shape of a banner, poster, interactive object that can used in the video game, and become familiar with the features and properties of the products advertised in this way. In time, users can develop strong beliefs and positive feelings about the product or brand, which can lead to purchase intention and actual acquisitions.

3.1 THE MODEL

The advertising competition model proposed in this paper is based on the Cournot oligopoly model with the following setup:

- There is a single type of advertising insertions that *n* firms insert into a certain online video game;
- The profit of firm *i* is a function of all the competitors' outputs.

Accordingly, assume sales q_i, conditioned by the effect of advertising x_i, can be expressed as:

$$q_i = ax_i - bx_i \sum_{j=1}^{n} x_j \qquad (1)$$

Where: *a* - is the initial effect on sales from a marginal unit of advertising, and

1. *b* - represents an immunizing effect on consumers from total advertising

The maximum volume of advertising that a video game can take is the volume at which sales become zero, i.e., when

$$q_i = 0 \Rightarrow \sum x_j = \frac{a}{b} \qquad (2)$$

The use of online video games for advertising purposes has an effect on consumers that is difficult to quantify for variables such as brand awareness, sympathy or sharing game experience, therefore in this paper it will be measured in final acquisitions of the products, meaning actual sales for the firms. The profits obtained by a firm will depend only on advertising amounts and, to focus only on advertising effect, we simplify by ignoring the effect of commodity price. If there are several competing commodities, the prices of all commodities will be taken as equal, and normalized to unity. Advertising has a unit cost c_i associated with it. To simplify, we consider all c_i equal so the profit that a firm obtains is:

$$\Pi_i = ax_i - bx_i \sum x_j - cx_i \qquad (3)$$

STRATEGIC GAME:

- ***players:*** *n* firms with rational comportment that buy advertising units in a certain online video game;
- ***the market:*** the online video game space;
- ***each firm's set of actions:*** set of all possible insertions that a firm can make in a video game;
- ***each firm's interest*** is represented by profits obtained by actual sales caused by advertising;

- ***decisions*** regarding the output (advertising quantity included into the video games) are made consecutively. Every firm bases his decision on the moves of the other competitors.

In the sequel, we analyse several particular cases.

3.2 MONOPOLY CASE

For a monopoly there is just one firm, say number 1; then quantity demanded and the profits obtained due to advertisements are:

$$q_1 = ax_1 - bx_1^2 \tag{4}$$

and

$$\Pi_1 = ax_1 - bx_1^2 - cx_1 \tag{5}$$

respectively.

The maximum advertising volume x_1 that the firm could insert in the online video game is reached when the marginal revenue equals to 0. This means that if firm 1 inserts another advertisement unit into the video game, another banner or branded item, it will no longer bring any additional income and the users will not buy any extra product.

Profits are maximal when the first derivative of the profit function is zero, i.e. when marginal revenue equals marginal cost. So:

$$\frac{\partial \Pi_1}{\partial x_1} = a - 2bx_1 - c = 0 \qquad \Rightarrow \qquad x_1 = \frac{a-c}{2b} \tag{6}$$

Unlike the case of oligopoly with several acting firms, there is no issue of dynamics or stability for the case of monopoly. The monopolistic firm hence has no need to modify the quantity $x_1 = \frac{a-c}{2b}$, unless anything, such as a parameter value, changes, given, of course that the second order maximum condition $\frac{\partial^2 \Pi_1}{\partial x_1^2} = -2b < 0$ holds, which it obviously does.

The monopoly firm, of course, only stays in business when it obtains, not only maximal, but a positive profit, i.e. when (5) with $x_1 = \frac{a-c}{2b}$ substituted is positive. This means $\Pi_1 = a\frac{a-c}{2b} - b\left(\frac{a-c}{2b}\right)^2 - c\frac{a-c}{2b} = \frac{(a-c)^2}{4b} > 0$, or, in plain words that the initial effect of advertising exceeds its unit cost. The possible alternative is a<c, for which $(a-c)^2 > 0$ holds. It is irrelevant that the advertising quantity according to (6) would be negative.

3.3 DUOPOLY CASE

If in the virtual space of an online video game where a firm is in monopolistic equilibrium, another firm enters, the advertising insertions state can be destabilised. For two competitors duopoly equilibrium might emerge instead.

The sales function for the 2 firms will be:

$$\begin{cases} q_1 = ax_1 - bx_1(x_1 + x_2) \\ q_2 = ax_2 - bx_2(x_1 + x_2) \end{cases} \tag{7}$$

And the profits:

$$\begin{cases} \Pi_1 = ax_1 - bx_1(x_1 + x_2) - cx_1 \\ \Pi_2 = ax_2 - bx_2(x_1 + x_2) - cx_2 \end{cases} \tag{8}$$

Maximizing profits:

$$\begin{cases} \dfrac{\partial \Pi_1}{\partial x_1} = a - 2bx_1 - bx_2 - c = 0 \\ \dfrac{\partial \Pi_2}{\partial x_2} = a - bx_1 - 2bx_2 - c = 0 \end{cases} \tag{9}$$

Solving for x_1, x_2 in (9), we find the reaction functions for the 2 firms:

$$\begin{cases} x_1 = \phi_1(x_2) = \dfrac{a-c}{2b} - \dfrac{1}{2}x_2 \\ x_2 = \phi_2(x_1) = \dfrac{a-c}{2b} - \dfrac{1}{2}x_1 \end{cases} \tag{10}$$

Both firms are active if their profits are positive. Otherwise, they would drop out of the market. As a negative reaction has no sense, we have the restrictions $\phi_1 > 0, \phi_2 > 0$ for the reaction functions:

$$x_i = \begin{cases} \dfrac{a-c}{2b} - \dfrac{1}{2}x_i, x_i \le \dfrac{a-c}{b} \\ 0, x_i \ge \dfrac{a-c}{b} \end{cases} \tag{11}$$

The Cournot equilibrium point is obtained at the intersection of the two lines defined by the system (10).

In basic linear algebra, the intersection of two straight lines in the plane defined by 2 linear equations

$$\begin{cases} a_1x + a_2y = a_3 \\ b_1x + b_2y = b_3 \end{cases} \tag{12}$$

can be the empty set, a point, or a line. If the equation system has a unique solution, the intersection of the two lines is a point, its coordinates satisfy both equations and can be found using substitution method. If the two equation system have no unique solution, either the lines are parallel and the system has no solution, or they are coincident and the system has an infinite solution- set.

By solving system (10) we find the volumes of advertising inserted in the online video game when there are 2 firms present in the same video game environment. To solve for the Cournot equilibrium point, we first add the reaction functions (10):

$$x_1 + x_2 = \frac{a-c}{b} - \frac{1}{2}(x_1 + x_2) \tag{13}$$

Moving to the left side all the $(x_1 + x_2)$ we have

$$\frac{3}{2}(x_1 + x_2) = \frac{a-c}{b}$$

Solving for the sum of advertising volumes

$$x_1 + x_2 = \frac{2}{3}\frac{a-c}{b} \tag{14}$$

Using x_1, x_2 obtained from (14) in (10) we find the Cournot equilibrium point:

$$\begin{cases} \overline{x_1} = \frac{1}{3}\frac{a-c}{b} \\ \overline{x_2} = \frac{1}{3}\frac{a-c}{b} \end{cases} \tag{15}$$

3.4 TRIOPOLY CASE

Consider that in a stable duopolistic system enters a third firm. Then, for 3 competitors, the equilibrium point might be destabilized. In this case the sales functions are:

$$\begin{cases} q_1 = ax_1 - bx_1(x_1 + x_2 + x_3) \\ q_2 = ax_2 - bx_2(x_1 + x_2 + x_3) \\ q_3 = ax_3 - bx_3(x_1 + x_2 + x_3) \end{cases} \tag{16}$$

and the profits

$$\begin{cases} \Pi_1 = ax_1 - bx_1(x_1 + x_2 + x_3) - cx_1 \\ \Pi_2 = ax_2 - bx_2(x_1 + x_2 + x_3) - cx_2 \\ \Pi_3 = ax_3 - bx_3(x_1 + x_2 + x_3) - cx_3 \end{cases} \tag{17}$$

Maximizing profits $\frac{\partial \Pi_i}{\partial x_i} = 0$ and solving for x_1, x_2, x_3 we obtain the reaction functions:

$$\begin{cases} x_1 = \phi_1(x_2 + x_3) = \frac{a-c}{2b} - \frac{1}{2}(x_2 + x_3) \\ x_2 = \phi_2(x_1 + x_3) = \frac{a-c}{2b} - \frac{1}{2}(x_1 + x_3) \\ x_3 = \phi_3(x_1 + x_2) = \frac{a-c}{2b} - \frac{1}{2}(x_1 + x_2) \end{cases} \tag{18}$$

On a market with three competitors we can again consider their actions in a dynamic system. If they take decisions based on "naive" expectation, they will move in each current period according to the previous moves of all their competitors. The iterative map in this case is:

$$\begin{cases} x_1' = \phi_1(x_2 + x_3) \\ x_2' = \phi_2(x_1 + x_3) \\ x_3' = \phi_3(x_1 + x_2) \end{cases} \tag{19}$$

Similarly to the duopoly case, we can find the equilibrium point by solving (18):

$$(x_1 + x_2 + x_3) = 3\frac{a-c}{2b} - \frac{1}{2}2(x_1 + x_2 + x_3) \Rightarrow$$

$$(x_1 + x_2 + x_3) = \frac{3}{4}\frac{a-c}{b} \tag{20}$$

Substituting $(x_2 + x_3), (x_1 + x_3), (x_1 + x_2)$ from (20) in (18) we find the Cournot equilibrium coordinates:

$$\begin{cases} \overline{x_1} = \dfrac{a-c}{4b} \\ \overline{x_2} = \dfrac{a-c}{4b} \\ \overline{x_3} = \dfrac{a-c}{4b} \end{cases} \tag{21}$$

Using the notations: $z_i = x_i - \overline{x_i}$ for the deviations from the Cournot equilibrium point like in the case of duopoly we obtain from (18), (19) and (21):

$$\begin{cases} z_1^{'} = -\dfrac{1}{2}(z_2 + z_3) \\ z_2^{'} = -\dfrac{1}{2}(z_1 + z_3) \\ z_3^{'} = -\dfrac{1}{2}(z_1 + z_2) \end{cases} \tag{22}$$

Using the notations:

$$\begin{cases} \omega_1 = z_1 - z_2 \\ \omega_2 = z_2 - z_3 \\ \omega_3 = z_1 + z_2 + z_3 \end{cases} \tag{23}$$

(22) becomes:

$$\begin{cases} \omega_1^{'} = \dfrac{1}{2}\omega_1 \\ \omega_2^{'} = \dfrac{1}{2}\omega_2 \\ \omega_3^{'} = -\omega_3 \end{cases} \quad \text{with the solutions} \quad \begin{cases} \omega_1 = \left(\dfrac{1}{2}\right)^t \omega_1(0) \\ \omega_2 = \left(\dfrac{1}{2}\right)^t \omega_2(0) \\ \omega_3 = (-1)^t \omega_3(0) \end{cases}$$

The Jacobian matrix of (22) in this case is:

$$\Im = \begin{Bmatrix} 0 & -\dfrac{1}{2} & -\dfrac{1}{2} \\ -\dfrac{1}{2} & 0 & -\dfrac{1}{2} \\ -\dfrac{1}{2} & -\dfrac{1}{2} & 0 \end{Bmatrix} \tag{24}$$

and the characteristic equation becomes $P(\lambda) = \lambda^3 - \dfrac{3}{4}\lambda + \dfrac{1}{4} = 0$,

with the roots $\lambda_{1,2} = \dfrac{1}{2}, \lambda_3 = -1$

As the value $\lambda_3 = 1$ falls at the limit of unit circle in the complex plane, the system is neutrally stable, and any perturbation would throw it into an endless oscillation. The system will be destabilized if the competitors become more than three (Theocharis,1959).

Following the same steps, if we calculate for a quadropoly we will find that the roots for the characteristic equation are $\lambda_{1,2,3} = \dfrac{1}{2}$ and $\lambda_4 = -\dfrac{3}{2}$. If they are limited by non-negativity constrains, as negative reaction have no sense, they will drop out of the market.

3.5 GENERAL CASE

If we generalize to n competitors, then the sales function is $q_i = ax_i - bx_i \sum x_j$ and the profits $\prod_i = ax_i - bx_i \sum x_j - cx_i$.

By maximizing all the profits $\frac{\partial \prod_i}{\partial x_i} = 0$ we can find the reaction functions of the firms as:

$$x_i = \frac{a-c}{2b} - \frac{1}{2}\sum_{j \neq i} x_j \tag{25}$$

It is convenient to introduce the definition $X_i = \sum_{j \neq i} x_j = X - x_i$, where $X = \sum x_j$. Then (25) can be written $x_i = \phi_i(X_i) = \frac{a-c}{2b} - \frac{1}{2}X_i$. Taking the sum of all equations we have $X = n\frac{a-c}{2b} - \frac{1}{2}(nX - X)$ and can solve for the total advertising volume

$$X = \frac{n}{n+1}\left(\frac{a-c}{b}\right) \tag{26}$$

The n firms being identical, they take an equal share of advertising, so

$$\overline{x_i} = \frac{1}{n+1}\left(\frac{a-c}{b}\right) \tag{27}$$

If there are n firms on the market and they act and react in accordance with the competitors' moves, we can consider their actions in a dynamical system. The iterative map is in this case:

$$x_i^{'} = \phi_i(X_i) = \frac{a-c}{2b} - \frac{1}{2}X_i \tag{28}$$

Using the notations: $z_i = x_i - \overline{x}_i$ for deviations from the Cournot equilibrium point and the definitions:

$$\begin{cases} \omega_1 = z_1 - z_2 \\ \omega_2 = z_2 - z_3 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \omega_i = z_i - z_{i+1} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \omega_{n-1} = z_{n-1} - z_n \\ \omega_n = z_1 + z_2 + \dots + z_n \end{cases} \tag{29}$$

we will see that the differences will be damped out, whereas the sum is constantly growing when the number of firms on the market in higher than 3. Equations (29) then become:

$$\begin{cases} \omega_1^{'} = \frac{1}{2}\omega_1 \\ \omega_2^{'} = \frac{1}{2}\omega_2 \\ \ldots\ldots\ldots\ldots \\ \omega_i^{'} = \frac{1}{2}\omega_i \\ \ldots\ldots\ldots\ldots\ldots \\ \omega_{n-1}^{'} = \frac{1}{2}\omega_{n-1} \\ \omega_n^{'} = -\frac{n-1}{2} \end{cases} \tag{30}$$

with solutions $\omega_i = \left(\frac{1}{2}\right)^t g\omega_i(0), i = 1,...,n-1$ and $\omega_n = \left(-\frac{n-1}{2}\right)^t \omega_n(0)$

The Jacobian matrix is:

$$\Im = \begin{bmatrix} \frac{\partial x_1^{'}}{\partial x_1} & \frac{\partial x_1^{'}}{\partial x_2} & \cdots & \frac{\partial x_1^{'}}{\partial x_n} \\ \frac{\partial x_2^{'}}{\partial x_1} & \frac{\partial x_2^{'}}{\partial x_2} & \cdots & \frac{\partial x_2^{'}}{\partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial x_n^{'}}{\partial x_1} & \frac{\partial x_n^{'}}{\partial x_2} & \cdots & \frac{\partial x_n^{'}}{\partial x_n} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{2} & \cdots & -\frac{1}{2} \\ -\frac{1}{2} & 0 & \cdots & -\frac{1}{2} \\ \cdots & \cdots & \cdots & \cdots \\ -\frac{1}{2} & -\frac{1}{2} & \cdots & 0 \end{bmatrix} \tag{31}$$

The characteristic equation in this case will have n-1 roots $\lambda_{1,\ldots,n-1} = \frac{1}{2}$, and one root $\lambda_n = -\frac{n-1}{2}$ so that, when there are more than 3 firms competing on a market, the last Eigenvalue will always fall out of the unit circle in the complex plane. In this case the system will be drawn away from the Cournot equilibrium point as it will expand continuously, or in other words will "explode".

This corresponds to the result that Theocharis (1959) found. By the way, his result was previously discovered by Palander (1939) twenty years earlier. Both Palander and Theocharis considered local stability only. In a global dynamic a linear system would just be damped out or explode. However, if we consider the non-negativity constrains occasionally discussed above, the global dynamics of identical firms would make them drop out every second time period (Cánovas, Puu, Ruiz, 2006).

CONCLUSIONS

The dynamic competition specific for today's markets made advertisers use new strategies for improving their brand image and obtain competitive advantages. One of these new strategies involves the use of video games for advertising purposes. The advantages are given by the specific attributes of the online environment and the additional entertainment that a video game offers to a player. The presence of a brand in the highly interactive environment of video games can bring serious advantages like a superior receptiveness of the advertising message, increased brand awareness and familiarity, long time relations with consumers, positive feelings, increased purchase intentions and actual sales.

The model proposed in this paper show that in a dynamic competition when multiple brands are present in the same video game, firms can obtain optimal advantages by reaching the equilibrium point. They can do this by choosing to act according to the actions of the other firms present in the game, respectively by adjusting their advertising volumes. The system equilibrium is conditioned bought by the total level of advertising and by the number of competitors. The advertising volume reaches its maximal level at a point where the consumer' mind is saturated by the quantity of advertising he is exposed to. At this level, any advertising effort will not bring additional effect on consumer.

The model also shows that the equilibrium of the competition can be maintained only if there are maximum three competitors. If there are more then three firms present in a particular video game none of them will obtain optimal benefits from the advertising insertions. The advertisements will have no effects on consumers who will maintain their initial knowledge, attitudes and preferences about those brands as before being exposed to the advertising messages, making the promotional campaign ineffective.

These aspects have important implications in strategic planning processes; a firm should choose a video game which contains a small number of advertisements or to decide over an exclusive presence in a video game, i.e., an advergame or an exclusive sponsorship of a video game.

Acknowledgment

This work was possible with the financial support of the Sectorial Operational Programme for Human Resources Development 2007-2013, co-financed by the European Social Fund, under the project number POSDRU/107/1.5/S/77946 with the title „Doctorate: an Attractive Research Career".

The author of this work during the mobility at Centre for Regional Science (CERUM), Umeå University, gratefully thanks for the hospitality and the help received during the staying.

REFERENCES

1. Cánovas, J.S., Puu, T., Ruiz, M. (2008) *The Cournot–Theocharis problem reconsidered,* Chaos, Solitons and Fractals No. 37, p. 1025–1039
2. Cellini, Roberto, Lambertini, Luca, Mantovani, Andrea (2008) *Persuasive advertising under Bertrand competition: A differential game,* Operations Research Letters 36 381–384
3. Cellini, R., Lambertini, L. (2003) *Advertising in a Differential Oligopoly Game*, Journal of Optimization Theory and Applications, Vol. 116, No. 1, pp. 61–81
4. Dirven, C.A.J.M., Vrieze, O.J. (1985) *Advertising Models, Stochastic Games and Myopic Strategies*, Operations Research, Vol. 34, No. 4, pp. 645-649, online: http://www.jstor.org/stable/170611
5. Dixit, A. (1979*) A model of duopoly suggesting a theory of entry barriers*, Bell Journal of Economics, 10(1): 20–32.
6. Erickson, G.M. (1985) *A Model of Advertising Competition*, Journal of Marketing Research, Vol. 22, pp. 297–304
7. Erickson, G.M. (1995) *Differential game models of advertising competition*, European Journal of Operational Research, No 83, pp. 431-438
8. Feichtinger, G. (1983) *The Nash Solution of an Advertising Differential Game: Generalization of a Model by Leitmann and Schmitendorf*, IEEE Transactions on Automatic Control, Vol. 28, p. 1044–1048.
9. Fershtman, C. (1984) *Goodwill and Market Shares in Oligopoly*, Economica, Vol. 51, pp. 271–282,
10. Friedman, L. (1957) *The Allocation of advertising expenditures*, presented at the Twelfth National Meeting of the Operations Research Society of America, Pittsburgh

11. Jorgensen, S. (1982) *A Survey of Some Differential Games in Advertising*, Journal of Economic Dynamics and Control, Vol. 4, p: 341–369
12. Leitmann, G., Schmitendorf, W.E. (1978) *Profit maximization through advertising: a nonzero sum differential game approach*, IEEE Trans. Automatic Control AC-23 (1978) 645–650.
13. Martin, S. (2002) *Advanced Industrial Economics,* Blackwell: Oxford.
14. Nerlove, M., Arrow, K.J. (1962) *Optimal Advertising Policy under Dynamic Conditions*, Economica, Vol. 29, pp. 129–142
15. Palander ,T.(1936) *Instability in competition between two sellers*, in: Abstracts of Papers Presented at the Research Conference on Economics and Statistics held by the Cowles Commission at Colorado College, Colorado College Publications, General Series No.208, Studies Series No. 21
16. Perloff J. 2008. *Microeconomics: Theory and Applications with Calculus*. Pearson Education: Boston, MA.
17. Sethi, S.P. (1977) *Optimal Advertising for the Nerlove–Arrow Model under a Budget Constraint*, Operations Research Quarterly, Vol. 28, pp. 683–693
18. Theocharis, R.D. (1959) *On the stability of the Cournot solution on the oligopoly problem*, Rev Econ Stud No. 27:133–4.
19. Vidale, M.L, Wolfe, H.B. (1957) *An Operations Research Study of Sales Response to Advertising*, Operations Research, Vol. 5, pp. 370–381
20. Winkler, T., Buckner, K. (2006) *Receptiveness of gamers to embedded brand messages in advergames: Attitudes towards product placement*, Journal of Interactive Advertising, 7(1)
21. Zheng, Y., Bar, T., Kaiser, H.M. (2009) *Generic Advertising in an asymmetric Cournot Oligopoly*, American Journal of Agricultural and Applied Economics, 92(3): 740–754