MAE- ALTERNATIVE METHOD OF MEASURING THE GLOBAL AVERAGE UNCERTAINTY OF INFLATION FORECAST IN ROMANIA

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Abstract:
In this paper I built forecasts intervals for the inflation rate in Romania, using the quarterly predicted values provided by the National Bank of Romania for 2007-2010. First, I used the historical errors method, which is the most used method, especially by the central banks. Forecast intervals were built considering that the forecast error series is normally distributed of zero mean and standard deviation equal to the RMSE (root mean squared error) corresponding to historical forecast errors. I introduced as a measure of economic state the indicator δ *-relativevariance of the phenomenon at a specific time in relation with the variance on the entire time horizon Then, I calculated the relative volatility in order to know the change that must be brought to the root mean squared error in order to take into account the state of economy. Finally, I proposed a new way of building forecasts intervals, when the date series follows an autoregressive process of order 1. In this case the length of forecasts interval is smaller and I got a slightly higher relative variance. I consider really necessary the building of forecasts intervals, in order to have a measure of predictions uncertainty, which is quantified by the National Bank of Romania using the prediction intervals based on a simple methodology. I calculated the forecasts intervals using MAE (mean absolute error), the indicator chose by National Bank of Romania and the MSE (mean squared error) indicator.*

Key words: forecasts intervals, historical forecasts errors, root mean squared error (RMSE), relative variance, uncertainty

JEL classification: C 32

INTRODUCTION

Economic forecasts are used for a certain purpose, mostly being related to the orientation in taking economic decisions. However, these forecasts are affected by uncertainty for which statistical measures of evaluation are used.

Public perception tends to associate to macroeconomic forecasts published by the government with the precision domain, with no prospect of uncertainty. The accompanying of forecasts with instruments for measuring the uncertainty provides autonomy to public environment involved in forecasts developing. The government uses various strategies to minimize the uncertainty. Krause demonstrates how risk management strategies provide recommendations on how to adapt to changing economic conditions. Uncertainty is based essentially on associating probabilities to future events verisimilitude (Krause, 2002).

Until the beginning of '50s, the economic phenomena were analyzed from the deterministic point of view, but the complexity of economic behaviour made necessary the stochastic concepts in describing the evolution of the economic processes and phenomena (Stancu and Mihail, 2005). Since predicting a variable by providing numerical values implies a high degree of uncertainty, the researchers have focused on the builing of intervals where the predicted value might appear with a certain probability.

All the institutions base their forecasts uncertainty on historical errors, but even in this case the studies based on this method of quantifing the uncertainty in literature are almost nonexistent, except those of Williams and Goodman (Knüppel, 2009).

Chatfield shows the necessity to accompany the predictions by forecasts intervals, which represent the uncertainty degree of variation. The probabilities of certain events can be given. Fair emphasizes that the possibility of an economic crisis should be specified within the forecast interval (Chatfield, 1993).

After a brief overview of the main achievements in literature related to the construction of prediction intervals, I built forecasts intervals for quarterly inflation rate predicted by the National Bank of Romania in 2007-2010 using the historical errors method, taking then into account the state of the economy. In addition, given that inflation rates series follows an autoregressive process of order 1, I proposed a new method for building prediction intervals. Finally, I compared the quarterly forecasting intervals on horizon 2007-2010 for inflation in Romania using MAE with those using MSE as synthetic indicators of forecasting error.

FORECASTS INTERVALS

The problem of building forecast intervals and the determination of distributions was approached quite late in the literature, notable works in this area being written by Cogley, Adolfson, Clark and Jore, Giordani and Villiani. The results showed an important conclusion: in order to build a forecast interval with a certain probability, the model has to include variances deviation in time.

Kjellberg and Villani numbered the advantages and disadvantages of both types of forecasts, the ones based on models and those built by the experts. Forecast methods based on models describe the complex relationships using endogenous variables by its transparence making easy the identification of mistakes that generated wrong predictions. The disadvantages are related to the difficulty of adapting the model to recent changes in the economy, as well as the too simple form of the models (Kjellberg and Villani, 2010).

Chatfield shows that forecast intervals are often too narrow not taking into account the uncertainty related to model specification, problem that is encountered also in the experts' assessment. Unlike the forecasts based exclusive on models, expert assessments modify immediately to any change of information related to the predicted phenomenon. Disadvantages in experts assessments are related just to the low degree of transparency, the difficulty of using many explanatory variables outside an explicit model. The way to build a forecasts interval is described by Granger, the retrospective presentation of the methods being done by Chatfield. Christoffersen explains how to evaluate these intervals while the methods for measuring forecasts density are introduced only in 1999 by Diebold, who extends them later for bivariate data (Chatfield, 1993).

Wallis is the first one who proposes tests for forecasts intervals, while Otrok and Whiteman, Robertson and Cogley introduce bayesian prediction intervals. Unlike other methods of building prediction intervals that are specified in literature, the Bayesian ones also analyze the impact of estimator error on interval. Stock and Watson specify the conditional distribution function for k-steps-ahead forecasts. Their approach is developed by Hansen, who built asymptotic forecasts intervals to include the uncertainty determined by the parameter estimator (Hansen, 2005).

BUILDING PREDICTION INTERVALS BASED ON HISTORICAL FORECAST ERRORS

The building of intervals taking into account the forecasts accuracy is an effective way to highlight the uncertainty that accompanies any forecast made. In the following, I used historical forecast errors to determine the forecast interval for inflation. I also used the projected inflation rates at the end of the year published by the National Bank of Romania for each quarter from 2007 to 2010. Forecast errors are calculated as the difference between expected value and the registered value. Forecast errors for each quarter are calculated by RMSE (root mean squared error). If we consider, $\hat{X}_t(k)$ the predicted value after k periods from the origin time t, then the error at future time (t+k) is: $e_t(t+k)$. Root Mean Squared Error (RMSE) is calculated

$$RMSE = \sqrt{\frac{1}{n}\sum_{j=1}^{n} e_X^2(T_0 + j, k)}\ .$$

Forecast intervals are built considering that the forecast error series is normally distributed of zero mean and standard deviation equal to the RMSE corresponding to historical forecast errors. For a probability of (1-α), forecast interval is calculated: $(X_t(k) - z_{\alpha/2} \cdot RMSE(k),\ X_t(k) + z_{\alpha/2} \cdot RMSE(k)), k = 1,...,K$

$X_t(k)$- punctual forecast for variable X_{t+k} at time t

$z_{\alpha/2}$- the α/2 quintile of standardized normal distribution.

The table below displays the RMSE and lower and upper limits of the forecast interval for inflation predicted by the central bank with a quarter before ('one-step-ahead ").

Table 1. The limits of the inflation rate forecasts intervals in Romania from 2007 Q1 to 2010 Q4 (based on historical forecasts errors)

Quarter	RMSE	Lower limit	Upper limit
2007 Q1	0,67	3,18	5,82
2007 Q2	0,51	3,31	5,29
2007 Q3	0,19	4,42	5,18
2007 Q4	1,99	0,79	8,61
2008 Q1	1,65	3,06	9,54
2008 Q2	2,36	1,57	10,83
2008 Q3	2,72	0,07	10,73
2008 Q4	2,51	-0,62	9,22
2009 Q1	0,77	4,49	7,51
2009 Q2	0,59	4,35	6,65
2009 Q3	0,11	4,88	5,32
2009 Q4	0,06	4,38	4,62
2010 Q1	0,43	3,35	5,05
2010 Q2	0,02	4,34	4,41
2010 Q3	0,27	7,24	8,30
2010 Q4	0,31	7,56	8,78

Source: calculations made using data from reports of inflation of National Bank of Romania between 2006-2010 - www.bnr.ro

The forecast intervals based on RMSE are independent of the state of the economy. Therefore, Blix and Sellin proposed the change of the method, so that the interval takes into account of changes in the economy, multiplying RMSE by a factor of uncertainty subjective chosen by the expert in forecasting (Blix and Sellin, 1998). Another approach uses, for the series of observations, a model in which time varies. Theseries of quarterly inflation rates follows an autoregressive AR process in which the series has a residual variance of stochastic type. It is assumed the hypothesis that errors are identically distributed and follows a standardized normal distribution. Then, the regression model can be written:

$ri = m + \sum_{k=1}^{K} \phi_k (ri_{t-k} - m) + \alpha_t \cdot e_t$, where α_t is the standard deviation of errors $\ln \alpha_t^2 = \ln \alpha_{t-1}^2 + \varepsilon_t$, where ε_t follows a normal distribution and $\ln \alpha_t^2$ is a random walk

We introduce a new statistical measure called the relative volatility or relative variance (variance of T moment in relation with the geometric mean of variances corresponding to the interval used to calculate RMSE), calculated by the formula: $\beta_T = \dfrac{\hat{\alpha}_T}{n^{-1} \prod_{t=t_1}^{t_2} \hat{\alpha}_t^{\frac{1}{n}}}$ t_1 and t_2 are the initial moment and the final one of the period for which RMSE is calculated, the time of the interval bounded of the two moments is: $n = t_1 + t_2 - 1$, and $\hat{\alpha}_T$ is a bayesian estimation.

The new intervals of variation of forecast values will be calculated as follows:

$(X_t(k) - z_{\alpha/2} \cdot \alpha_t \cdot RMSE(k), X_t(k) + z_{\alpha/2} \cdot \alpha_t \cdot RMSE(k)), k = 1,...,K$. The relative volatility is 0,279.

Relative volatility of Q4 of 2010 was 1.279, which means that a 62.1% decrease in the value of RMSE is necessary to take into account the changes in the economy.

THE PROPOSAL OF A NEW WAY TO BUILD FORECAST INTERVALS APPLICABLE IN ROMANIAN BANKS

Between 2007-2010 inflation rates calculated at the end of the quarter may be represented by an AR process of order 1 (AR (1)). To determine the interval of variances of BNR predictions taking into account the state of the economy in each of the periods for which data were recorded, the coefficient which multiplies RMSE is calculated in different way than that recommended in the literature. Inflation is modeled in 2007-2010 as: $r\_\inf_t = 6,917 + 0,714 \cdot r\_\inf_{t-1} + e_t$

For an AR process ($X_t = \varphi_1 \cdot X_{t-1} + e_t$), the variance is: $\text{var}(X_t) = \frac{\sigma_e^2}{1+\varphi_1^2}$, where σ_e^2 – AR process error variance.

The variance of inflation is: $\text{var}(r\_\inf) = \frac{\sigma_e^2}{1+0,714^2} = \frac{1,232}{1,509} = 0,816$

I introduce as a measure of economic state the indicator δ-relativevariance of the phenomenon at a specific time in relation with the variance on the entire time horizon, which for T moment is calculated as: $\delta_T = \frac{[e_T - E(e_t)]^2}{\text{var}(r\_\inf)} = 0,339$

Table 2. The limits of the inflation rate forecasts intervals in Romania from 2007 Q1 to 2010 Q4 (based on own method)

Quarter	e_t	$[e_t - E(e_t)]^2$	δ_t	RMSE	**Lower limit**	**Upper limit**
2007 Q2	-0,921	0,849	1,040	0,507	3,267	5,333
2007 Q3	0,307	0,094	0,116	0,193	4,756	4,844
2007 Q4	1,149	1,320	1,618	1,993	-1,621	11,021
2008 Q1	1,195	1,428	1,750	1,653	0,629	11,971
2008 Q2	0,905	0,819	1,003	2,363	1,552	10,848
2008 Q3	0,029	0,001	0,001	2,720	5,394	5,406
2008 Q4	-0,967	0,934	1,145	2,510	-1,333	9,933
2009 Q1	-0,071	0,005	0,006	0,770	5,991	6,009
2009 Q2	-0,722	0,521	0,639	0,587	4,765	6,235
2009 Q3	-1,336	1,785	2,188	0,113	4,614	5,586
2009 Q4	-0,980	0,961	1,177	0,063	4,354	4,646
2010 Q1	-0,603	0,364	0,445	0,434	3,817	4,575
2010 Q2	-0,923	0,852	1,044	0,017	4,342	4,412
2010 Q3	2,410	5,808	7,118	0,270	3,999	11,535
2010 Q4	0,526	0,277	0,340	0,311	7,960	8,374

Source: calculations made using data from reports of inflation of National Bank of Romania between 2006-2010 - www.bnr.ro

In this case, I obtained a relatively large variance, which means that it is necessary a decrease of RMSE value with 66.1% if one takes into the state of the economy in the last quarter of 2010.

THE METODOLOGY USED BY NATIONAL BANK OF ROMANIA AND AN ALTERNATIVE TO IT

Providing an evaluation of uncertainty is related to the effectiveness with which an institution fails to influence the economic activity. The methodology used by BNR is a simple one, like measure of global medium uncertainty for the rate on inflation based on its macroeconomic short-term forecast model is used the mean absolute error (MAE-mean absolute error). This synthetic indicator includes all effects of unanticipated past shocks that led the deviation of the expected values from the registered ones. Based on this type of error prediction, forecasting intervals are built, BNR numbering several advantages of its methodology:

- it considers all the previous shocks that have affected the rate of inflation;
- it determines a classification of the deviations from the actual values in the history of projections: deviations that determined an overestimation of the projected inflation and deviations that generated an underestimation;
- the methodology excludes any arbitrary assumption about the action of individual risk factors;
- it allows the adjustment of intervals of uncertainty, so that they reflect the assessments of different agents regarding the magnitude of the future uncertainty in relation with the one of previous periods.

Unlike the RMSE indicator, the indicator for forecasting error MAE is less sensitive to large prediction errors. If the dataset is small MAE is recommended, but the most institutions use RMSE as its unit of measurement is the same as the one of the indicator which is calculated. RMSE is always at least equal to the MAE. Equality occurs if the errors have the same magnitude. The difference between the MAE and the RMSE is higher, the greater the variability of the data series. RMSE is affected by generalized variance, the interpolation, the errors in the phase and by the presence of outliers.

Forecasts intervals are calculated, according to the simple general methodology:

$$(X_t(k) - z_{\alpha/2} \cdot \alpha_t \cdot RMSE(k), X_t(k) + z_{\alpha/2} \cdot \alpha_t \cdot RMSE(k)), k = 1,...,K \ .$$

If RMSE is replaced by other indicators, MAE (mean absolute error) or MSE (mean squared error) other intervals are built.

Mean absolute error are calculated as: $MAE = \frac{1}{n}\sum_{j=1}^{n} | \ e_X(T_0 + j, k) \ |$ and mean squared error is: $MSE = \frac{1}{n}\sum_{j=1}^{n} e^2{}_X(T_0 + j, k)$.

The new forecasts intervals are:

$$(X_t(k) - z_{\alpha/2} \cdot MAE(k), X_t(k) + z_{\alpha/2} \cdot MAE(k)), k = 1,..., K$$

$$(X_t(k) - z_{\alpha/2} \cdot MSE(k), X_t(k) + z_{\alpha/2} \cdot MSE(k)), k = 1,..., K$$

Table 3. Quarterly forecasting intervals for inflation in Romania on horizon 2007-2010 (calculated using the historical errors method, ex-post technical)

	il(MAE)	sl(MAE)	il(MSE)	sl(MSE)
2007Q1	2,641	6,359	0,787	7,539
2007Q2	2,441	6,159	1,414	6,679
2007Q3	2,941	6,659	4,266	5,527
2007Q4	2,841	6,559	1,902	9,492
2008Q1	4,441	8,159	1,032	13,221
2008Q2	4,341	8,059	-0,263	15,027
2008Q3	3,541	7,259	-0,195	13,715
2008Q4	2,441	6,159	1,273	9,837
2009Q1	4,141	7,859	3,051	9,719
2009Q2	3,641	7,359	3,891	7,696
2009Q3	3,241	6,959	4,553	5,533
2009Q4	2,641	6,359	3,123	5,940
2010Q1	2,337	6,055	2,950	5,876
2010Q2	2,518	6,236	2,602	6,135
2010Q3	5,907	9,626	3,394	11,869
2010Q4	6,308	10,027	3,089	12,935

Processing based on data from the BNR and the INS (www.bnr.ro , www.insse.ro) (il (MAE), il (MSE) and sl (MAE), sl (MSE), the lower respectively the upper forecasting interval limit, interval calculated using MAE, respectively MSE)

I built quarterly forecasting intervals on horizon 2007-2010 for inflation in Romania using MAE and MSE as synthetic indicators of forecasting error. I noticed that the length of intervals based on MSE is lower, so the uncertainty is higher.

CONCLUSIONS

Based on data of inflation forecasts provided quarterly by the Central Bank, forecast intervals were built using the method of historical forecast errors. For Romania, when inflation rates follows an AR (1), I have improved the technique of building forecast intervals taking into account the state of the economy in each period for which data were recorded. I recommend the use of interval forecasts by the National Bank of Romania based on MAE or MSE, in order to have forecasts accompanied by an objective degree of uncertainty, fact that improves the decisional process, because the uncertainty is lower.

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