ECONOMIC MODELS THAT LEAD TO LINIAR PROGRAMMING PROBLEMS

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Abstract:
In a modern market economy a manager for technical management of activities - economic prosperous must know optimization theory alongside other modern techniques such as computer science, analysis system etc. Problems optimization helps us to determine an optimum solution if it exis ts. We observe that the correct formulation optimization problems is important. Linear programming is used to determine the best of resources to achieve a minimum c ost or a maximum benefit. In this article are presented some economic models that lead to problems of linear programming. On the basis of solving these problems lies Simplex method. The first step in solving such problems is mathematical modeling, namely respect for the general form of mathematical model.

Keywords: *linear programming, depending end system restrictions, the general form of a linear programming problems.*

I. GENERAL CONSIDERATIONS REGARDING THE PROBLEM OF OPTIMIZATION

The economic modelling - mathematically represented by a linear programming problem contains a number of variables and parameters. Some of these variables are known, called variable entry, and others are unknown, called vari able output. Interconditions, links between system components restrictions transposing into the mathematical model of functional relationships, namely equations and / or inequations. The model is a mathematical function objective which links various variables and measuring a specific performance.

The goal is to model variables determine output depending on variables entrance so that it met performance criteria, that is to be resolved system. These problems with the conditions and a logical algorithm can deal with the computer.

Broad issues of a mathematical programming is:

$$[\max / \min]\, f(x_1, x_2, \ldots, x_n) \tag{1}$$

$$g_i(x_1, x_2, \ldots, x_n) \le / \ge / = 0, \quad i = \overline{1,m} \tag{2}$$

$$X = (x_1, x_2, \ldots, x_n)^t \in D \subset \mathbb{R}^n \tag{3}$$

where:

- relationship (1) represents the objective function, efficiency or purpose and function depends on variables decision;
- relationship (2) represents the restrictions on the optimization problem expressed with decision variables and is composed of conditions to be met when determining the values of variables decision.

A linear programming problem consists of two distinct parts:

- a number of restrictions linear:

$$\begin{aligned} & g_i(x_1, x_2, \ldots, x_n) \le b_i, \quad i = \overline{1,r} \\ & g_i(x_1, x_2, \ldots, x_n) = b_i, \quad i = \overline{r+1,s} \\ & g_i(x_1, x_2, \ldots, x_n) \ge b_i, \quad i = \overline{s+1,m} \\ & x_j \ge 0, \quad j = \overline{1,n} \end{aligned} \tag{4}$$

expressing the problem;

- a linear function, $f(x_1, x_2, \ldots, x_n)$, the objective of the problem, namely maximizing or minimizing it.

They say that we have a linear programming problem if all the f unctions g_i of the system restrictions (2) function and efficiency f are linear, and the variables x_i first occur in power and decision variables.

Starting from economic examples, will introduce the concept of linear programming problem which consists in finding an optimum functions under certain conditions. This problem may occur under various forms, where the restrictions may be a lot f inished or not, or not bordered solutions admissible. Therefore, plays an important role solutions basic admissible and optimal solutions.

Linear programming available at present, a treatment unit and has applications in various fields of science and social study economic phenomena. Linear programming is one of the main tools of economic analysis, as incorporated ideas Maximization, minimize or determination of a point "şa".

See that the mathematical models of economic problems associated presents some similarities, so they can be incorporated into a general model.

Broad a model of linear programming problem is:

$$[\min / \max]\ f(x_1, x_2, \ldots, x_n) = \sum_{j=1}^{n} c_j \cdot x_j \tag{5}$$

$$\begin{cases} \sum_{j=1}^{n} a_{ij} \cdot x_j \le b_i,\ i = \overline{1, r} \\ \sum_{j=1}^{n} a_{ij} \cdot x_j = b_i,\ i = \overline{r+1, s} \\ \sum_{j=1}^{n} a_{ij} \cdot x_j \ge b_i,\ i = \overline{s+1, m} \end{cases} \tag{6}$$

$$x_j \ge 0,\ j = \overline{1, n} \tag{7}$$

where:
- relationship (5) represents the objective function of efficiency or function end;
- relationship (6) represents the restrictions on the optimization problem;
- relationship (7) represents the nenegativity variables and ensure the achievement of a feasible solution in terms of economic logic.

In the Romanian economy in recent years several changes have occurred both on the social, political and economic. Economy represents a complex and dynamic system, so operators of small, medium and large are in a continuous adjustment to the requirements of market economy, where the needs and desires of consumers for products and services are without limit. All these programs have a single purpose such as: a program produced by the big profits with little expense or in a short time.

To determine the minimum cost and maximum benefits will use optimization theory. Optimization theory is used if a causal relationship stoc hastic and consists inside set of methods that help us to take decisions occur when several factors influence. The main t heory that characterize yourself are:

- research systems that are organized influenced by various factors;
- rationalization decisions, when you know certain scientific data and analysis;
- the application of scientific methods that show the links of interdepend ence, and bring them to a form of mathematics known when certain weights for each item or factor.

This theory encompasses minimize problems and maximize.

Examples will be presented to show three elements that characterize a model of linear programming problem: a linear function in all the arguments to be minimized or maximized, a system of restrictions linear format of equality or inequality not strict and conditions imposed nenegativity model variables.

II. ECONOMIC MODELS THAT LEAD TO LINEAR PROGRAMMING PROBLEMS

II.1. The issue concerning the optimal allocation of tasks of production

In an enterprise, different departments may produce different with the same efficiency products, besides subjective conditions that determine the effectiveness, May and acting objective factors which the organization responsible for scientific production must take into account in order to achieve economic optimum. We will use notations following:

- A_i - represents the products manufactured, $i=\overline{1,m}$;
- B_j - represents producing polling, $j=\overline{1,n}$;
- a_{ij} – represents coefficients technological;
- c_{ij} - represents the price of a unit cost of product A_i made in section B_j;
- b_i - represents the planned quantity of the product A_i;
- t_j - represents working time available in department B_j, or the total number of workers, or the total equipment available in department B_j etc.;
- x_{ij} - represents quantity of the product A_i what will be done in the department B_j, so that the total expenditure to be minimal.

To solve such a problem must to build first mathematical model of the problem of optimal allocation of production tasks. Mathematical model is:

$$[\min] f(x)=\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\cdot x_{ij}$$

$$\begin{cases} \sum_{j=1}^{n} x_{ij}=b_i, & i=\overline{1,m} \\ \sum_{i=1}^{m} a_{ij}\cdot x_{ij}\le t_j, & j=\overline{1,n} \end{cases} \qquad (8)$$

$$x_{ij}\ge 0,\quad i=\overline{1,m},\quad j=\overline{1,n}$$

II.2. The issue regarding the use of rational investment

Suppose that for a particular branch of production know:

- S - represents the total amount that can be invested in various activities;
- c_j - represents the benefit provided by investment in business $j, j=\overline{1,n}$;
- S_1 - represents the minimum amount to be consumed by activities 4,5,6;
- investment activity j = 1,2,3 must not exceed the amount of investment in the rest of the activities;
- investment activity j = 2 , the lack of technical conditions, may not exceed p% of the total activity.

It is required to determine the amount x_j to be invested in each activity, so that the amount allocated to ensure the achievement of a total maximum benefit.

To solve such a problem must to build first mathematical model of the problem regarding the use of rational investment. Mathematical model is:

$$[\max] f(x) = \sum_{j=1}^{n} c_j \cdot x_j$$

$$\begin{cases} \sum_{j=1}^{n} x_j = S, \\ x_4 + x_5 + x_6 \geq S_1, \quad S_1 \leq S \\ x_1 + x_2 + x_3 \leq \sum_{j=4}^{n} x_j \\ x_2 \leq \dfrac{p \cdot S}{100} \end{cases} \tag{9}$$

$$x_j \geq 0, \quad j = \overline{1,n}$$

II.3. The problem of determining the structure plan production units design

Whether an enterprise in which different sub unities performing the same work with different costs and consumption of a different time, as a result of differentiation in terms of experience, equipping and technical material, preparing designers. Not ation:

- A_i - represents the work of the design plan institution, $i = \overline{1,m}$;
- B_j - represents subunit design, $j = \overline{1,n}$;
- a_{ij} - coefficients technological means, representing the number of hours required to design a performing works by A_i design subunit B_j;
- c_{ij} - represents the cost price of paper A_i done B_j ;
- b_i - represents the amount of work planned A_i ;
- t_j - represents the number of hours of design that has subunit design B_j;
- x_{ij} - represents part of the work A_i to be performed by B_j so that production costs are minimal (x_{ij} unknown).

To solve such a problem must to build first mathematical model of the problem of determining the structure plan production units design. Mathematical model is:

$$[\min] f(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \cdot x_{ij}$$

$$\begin{cases} \sum_{j=1}^{n} x_{ij} = b_i, \quad i = \overline{1,m} \\ \sum_{i=1}^{m} a_{ij} \cdot x_{ij} \leq t_j, \quad j = \overline{1,n} \end{cases} \tag{10}$$

$$x_{ij} \geq 0, \quad i = \overline{1,m}, \quad j = \overline{1,n}$$

II.4. The issue on the sale of goods

Suppose that a business unit wants to set plans for sellin g a certain period of time. The conditions under which the disposal are characterized below.

- P_i - represents the type of merchandise i, $i = \overline{1,n}$;
- a_i - represents the lower limit of the plan of selling goods P_i during considered ($i = \overline{1,n}$);
- b_i - represents the upper limit of the plan of selling goods P_i during considered ($i = \overline{1,n}$);

- c_i - represents the benefit secured a unit of merchandise P_i expressed in currency units;
- d_i - means necessary storage space (warehouses, shelves etc.) for a unit of merchandise P_i;
- t_i - represents the time required for the dissolution of a unit of merchandise P_i;
- d - represents the total capacity of the space, storage;
- t - represents the total sales of goods.

To solve such a problem must to build first mathematical model of the problem on the sale of goods. Notation x_i represents the quantity of goods disposed of P_i for the benefit of total gross f is the maximum. Mathematical model is:

$$[\max] f(x) = \sum_{i=1}^{n} c_i \cdot x_i$$

$$\begin{cases} \sum_{i=1}^{n} d_i \cdot x_i \le d \\ \sum_{i=1}^{n} t_i \cdot x_i \le t \\ a_i \le x_i \le b_i, \quad i = \overline{1,n} \end{cases} \tag{11}$$

$$x_i \ge 0, \quad i = \overline{1,n}$$

II.5. The problem of storing the goods

Whether a wholesale commercial enterprise that handles the purchase and distribution of goods trade organizations subordinate. Assume that this undertaking is a number m of deposits they note D_j ($j = \overline{1,m}$) and that is subordinate p commercial organizations noted C_k ($k = \overline{1,p}$). Whether, as well:

- P_i - represents sorts of goods $i = \overline{1,n}$;
- a_{ij} - represents coefficients technological (area occupied by a unit of cargo P_i total capacity of the store D_j);
- b_{ik} - represents the quantity of goods P_i necessary trade organization C_k ($i = \overline{1,n}$; $k = \overline{1,p}$)
- C_{ijk} - represents a unit cost of transport of goods P_i ($i = \overline{1,n}$) stored in the center D_j ($j = \overline{1,m}$) the trade organization C_k ($k = \overline{1,p}$)
- x_{ijk} - represents the quantity of goods P_i stored in the center D_j and transported to commercial organization C_k.

We want to determine the quantities x_{ijk} that transport costs are minimal.

To solve such a problem must to build first mathematical model of the problem of storing the goods. Mathematical model is:

$$[\min] f(x) = \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{p} C_{ijk} \cdot x_{ijk}$$

$$\begin{cases} \sum_{i=1}^{n} a_{ij} \sum_{k=1}^{p} x_{ijk} \le 1, \quad j = \overline{1,m} \\ \sum_{j=1}^{m} x_{ijk} = b_{ik}, \quad i = \overline{1,n}, \quad k = \overline{1,p} \end{cases} \tag{12}$$

$$x_{ijk} \ge 0, \quad i = \overline{1,n}, \quad j = \overline{1,m}, \quad k = \overline{1,p}$$

III. CONCLUSION

In a modern market economy where the complexity of human activity grows, managers of a company, according to existing resources, in the attainment of a minimum cost of resources used and also achieve a maximum benefit. Managers must also meet the proposed objectives, namely performance management, based on existing resources. Quantities optimal allocation of resources to achieve a minimum cost is one of the issues most important economy, which is based on linear programming. This will analyze a problem with a practical example, so the results can be interpreted in terms of their practical and could be carried out an alyses of various sensitivity optimal solution. In the same way you can solve other more complicated patterns similar to the one shown.

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