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EXPLORING DETERMINISTIC CHAOS IN ECONOMIC SYSTEMS

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Abstract:
The term "chaotic" is used to describe the aperiodic behavior of an apparently random system. Behind the apparent random dynamics lies in fact the deterministic character of the system whose behavior is completely determined by the equations describing the deterministic chaotic dynamics. Chaos in proven to occur generally in complex nonlinear systems. Nonlinearity introduces in fact a better understanding of the complex natural phenomena. Nonlinear dynamics consists of a set of tools and concepts (period doubling, bifurcations, initial conditions sensitivity, attractors, phase space, phase portrait) allowing to analyze the dynamics generated by nonlinear processes. The paper presents the main ideas of the chaos theory and identifies the main papers that identified (or not) the presence of chaotic dynamics in different areas.

Key words: chaos, nonlinear dynamics, chaotic systems, sensitivity to initial conditions, fractals, catastrophe theory, complexity economics,

JEL classification: G00, M20, C02, C40

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1. INTRODUCTION

At the present moment, two ways of approaching the behavior of the complex economical systems are the most appealing: the complexity economics theory and the chaos theory. The complexity economics theory studies how self-organizing nonlinear complex systems are able to stay away from the extremes of possible behavior regimes: randomness and order. The edge of chaos is another concept stating that there is an intermediate space between order and disorder. Kaufman created a model that analyzes the fitness of an organism as the result of interacting genes. The NK model studies N entities linked through K connections and found out that the transition between order and disorder is the result of varying the number of connections. When K is too high, the disorder degree in the network makes the model unusable, and any stimulus propagates itself in the network leading to constant instability, but when K is low, the entities are too isolated, any perturbation are influencing few other entities, if any. (Kauffman, 1989)

Until the occurrence of chaos theory, it was considered that an intricate, irregular and complex behavior was random and unpredictable. Once the chaos theory emerged, new tools appeared and a new world became visible with new approaches being applicable. The main advantage of the chaos theory, from economic perspective, was the ability to produce models whose outputs were very close to the oscillations observed in the real economic environment.

There are many definitions of chaos, two of those are capturing the essence of the concept without appealing to sophisticated details: "chaos is the rediscovery that calculus does not has infinite power" (Baranger, 1995) and "chaos means order with no predictability" (Werndl, 2009). It has been proved that chaotic system are predictable due to the determinism existing in many complex nonlinear systems.

The first wrapping up of the determinism principle was stated by Pierre Laplace, statement that is known under the name of the "Laplace Demon". Laplace's idea was that the past influences the configuration of the future. "We may regard the present state of the universe as the effect of the past and the cause of its future. An intellect which at any given moment knew all of the forces that animate nature and the mutual positions of the beings that compose it, for such an intellect nothing could be uncertain and the future just like the past would be present before its eyes" (Laplace, 2011).

Different approaches of economic models were proposed over time. Some of them borrowed concepts and tried to discover similarities with notions and theories belonging to other fields of research, for instance thermodynamics. It has been observed that the relationships between pressure and volume in a thermodynamic system seems to be very similar to the connection between the price and volume of trades in an economic system. (Faginni, 2012)

The majority of economic models are considering that the competitive markets is an environment in which agents are interacting using as common language the prices that are set on the basis of supply and demand principle. Buying and selling are impersonal activities that are emerging taking into account the information transmitted by the prices of the assets. Agents are interpreting the information and then proceeds accordingly. The interactions among agents influence the overall (macroeconomic) behavior.

The Walrasian general equilibrium model the equilibrium is achieved solely by the prices system. The post Walrasian equilibrium model states that the future cannot be predicted, not even in probabilistic terms. The agents have insufficient information and limited ability to extract knowledge form the information available to them so that they interact having their own understanding of how the future will be. (Gintis, 2007)

2. COMPLEXITY, CATASTROPHE AND FRACTALS

Complex systems can evolve towards steady states or multiple equilibrium states, where systems can sometimes become stuck, requiring large shocks to switch to another regime. Inflection points may occur that cause a sudden change in behavior, the emergence of oscillations of chaotic or stochastic dynamics between which, until the emergence of chaos theory, there was no distinction.

2.1. COMPLEXITY ECONOMICS

Complexity economics studies economics from the perspective of complex systems. The economy is considered as a complex system that is made up of several subsystems, which are in turn complex with overlapping boundaries. The theorists in this field believe that the natural state of the economy as a system is disequilibrium (Arthur, 2013). The economy is constantly evolving and changing as a result of two fundamental issues: uncertainty and technological innovation. The concept of uncertainty was formulated at the beginning of the 20th century, which leads to a continuous adaptation to the economic reality and a continuous abandonment of decision-making strategies, while technological progress opens new directions of evolution, introducing even more variables and uncertainty generators and further disrupting the overall movement of the system.

2.2. CATASTROPHE THEORY

Considering continuous process, a slow change in one variable influencing the process will cause a slow change in another variable. In a discontinuous process, sudden changes in one variable result from a slow change in another.

The mathematician Rene Thom set out to create a mathematical apparatus which he used to attempt to predict the evolution of discontinuous processes, thus underpinning catastrophe theory.

He theorizes that sudden changes in the evolution of discontinuous systems can lead to a catastrophe, i.e. a completely different behavior from that of the previous time. Although the number of discontinuous processes in nature is virtually infinite, Thom found that it can be included in a certain typology based on a graphical representation of the evolution of the variables involved in the occurrence of catastrophes. Seven types of graphical representations (for processes involving four variables) can thus emerge: fold, spike, butterfly, wave, hair, reservoir and swallowtail. The values of the variables for which the catastrophe occurs are called the catastrophe-set. Catastrophe theory can be considered as a particular case of bifurcation theory.

Although initially generating general enthusiasm in academia, catastrophic theory has been placed in a shadow, the main drawback being that, in order to be studied using this theory, the system must be relatively simple, with a small number of variables involved in its evolution.

The main criticisms of the theory centered along the following lines:
- It relies exclusively on quantitative methods;
- Inadequate quantification in some applications;
- It uses particular mathematical models.

An early application of catastrophe theory in economics was the attempt to explain financial bubbles and crises (Zeeman 1974), but the best-known application of catastrophe theory was in analyzing the business cycle (Varian, 1979), proposing a model of form:

$$\frac{Dy}{Dt} = s(C(y) + Y(i,k)) - y$$

$$\frac{Dk}{Dt} = I(y,k) - I_0 \qquad (1)$$

$$C(y) = cy + D$$

where y is the national income, k is the long-run capital stock, C(y) is the consumption function, I(y,k) is the gross investment function, I_0 is the level of replacement investment, and s is the speed of the adjustment parameter considered fast relative to capital market movements.

Catastrophe theory has found its application in different domains such as: stock market movements, food riots, and the emergence of riots in detention centers (Poston, 2012).

With the decline of catastrophe theory (mainly due to criticisms coming from some researchers, criticisms not always justified) other methods for modeling dynamic discontinuities in economic models (Skiba point analysis in multiple equilibrium systems) have been used (Rosser, 2007).

2.3. FRACTALS

The notion of fractal was first used by Benoit Mandelbrot (1985) who studied the mathematical basis of nature patterns and refers to geometric objects characterized by details at an arbitrary scale, a certain degree of "self-similarity" and fractal Haussdorf dimension. The problem of defining more accurate measurement criteria other than Euclidean dimensions (length, volume, area) to quantify the geometric properties of fragmented objects was addressed long before the notion of fractal became popular. Thus, today, there are many criteria that allow the measurement of qualities that have no clear definition (such as the roughness of an object). Fractal structures cannot be described using Euclidean geometry. In order to measure the complexity of a fractal, a quantity is needed that will actually measure the difficulty of identifying some of its qualities. This quantity is called the fractal dimension.

Fractals are geometric structures that exhibit a particular type of complexity, which increases as the scale of visualization of the object increases. In other words, the more you look at the structure, the more you discover details which in turn contain even more complicated repeating structures that are identical to the whole, a characteristic of most fractals called self-similarity.
Fractal structure is found in abundance in nature in the form of plants, landforms, rivers, clouds, vegetables, etc. With the deepening of research in this field, some of the scientific community affirmed that the universe has a fractal structure that is found in both macroscopic and microscopic structures.

The applications of fractals are surprising firstly in their number and secondly in their ingenuity, depth and implications. Thus, Microsoft released the first edition of the Microsoft Encarta encyclopedia (1992) containing thousands of articles, 7000 photos and 800 color maps, all stored in less than 600MB thanks to the application of fractal theory in data compression. The basic idea behind this kind of compression, called fractal data compression, is to find each generating a fragment of the overall image.

Fractals are also present in the structure of some organs of the human body (lungs, brain, mitochondrial membrane), in the evolution and fluctuations of the market and stock exchanges. An understanding of fractal structure leads to the creation of models that will make it possible to understand the overall "functioning" of the system under study.

The study of chaos using fractals aims to predict patterns in dynamic systems that appear apparently unpredictable.

Figure 1. Natural fractal structure. Sunrise over the Himalayas
Source: Sprott, 2000.

Another very interesting idea was raised: the structure of the universe is partially fractal (Iovane et al., 2004).

3. CHAOS THEORY

Chaos and the set of concepts related to the dynamics of systems and their modeling by differential equations, called chaos theory, is closely related to the notion of nonlinearity. Nonlinearity, in turn, implies the loss of causal correlation between the disturbance and its propagated effect over time.

Chaos theory includes a set of notions, techniques and algorithms that attempt to explain the complex nonlinear dynamics of a system in terms of its behavior in phase space. In addition to a whole arsenal of notions coming from mathematics, physics or systems theory, there are also various concepts specific to this field: chaos boundary, low dimensional chaos, hyper chaos, deterministic chaos or non-deterministic chaos.

The edge of chaos is a transitional regime of maximum complexity, situated between order and chaos, which is assumed to be characteristic of complex, evolving systems capable of continuous reconfiguration and adaptation. This is the case of the economic system, ecosystems and human societies capable of functioning between regimes characterized by order or absolute disorganization. With regard to human society, the concept of the "black swan" has been formulated (Nassim, 2019), a rare, unpredictable, unpredictable event that is impossible to anticipate on a historical basis and that changes society and the course of history.

The identification of the occurrence and manifestation of chaos in the dynamics of a system is carried out by visual tests (recurrence diagrams, bifurcation diagrams, Poincare diagrams), numerical indicators (Lyapunov exponent, correlation dimension, Hausdorff dimension, Kolmogorov entropy) or by taking into account the results obtained by applying both categories of tools

Nonlinear analysis is based on Takens' state space reconstruction theorem. It aims at state space reconstruction based on the concept of topological equivalence. When working in a reconstructed low-dimensional phase space one aims at reproducing the essential features of the original dynamics without necessarily knowing the equations of the dynamical system that generated the time series under study.

The concept of topological equivalence allows the study of geometric objects of small size, which will provide information about the original dynamics. Topological equivalence and the low-dimensional attractor led to a major concept called capacity dimension. This size allows on the one hand the characterization of the attractor and on the other hand it allows the differentiation between deterministic chaos and random behavior.

Chaos theory or nonlinear deterministic dynamic system theory made a clear separation between random and deterministic chaotic system. The difference between chaos and randomness is that, in the case of chaotic behavior we don't know all the values to an infinite degree of precision. Randomness implies that we can't understand the synergic effect of all the variables influencing the complex system evolution. A random system has no structure but a chaotic system has a hidden one that can be revealed using visual instruments (phase space plots, Poincare maps, power spectra, etc.)

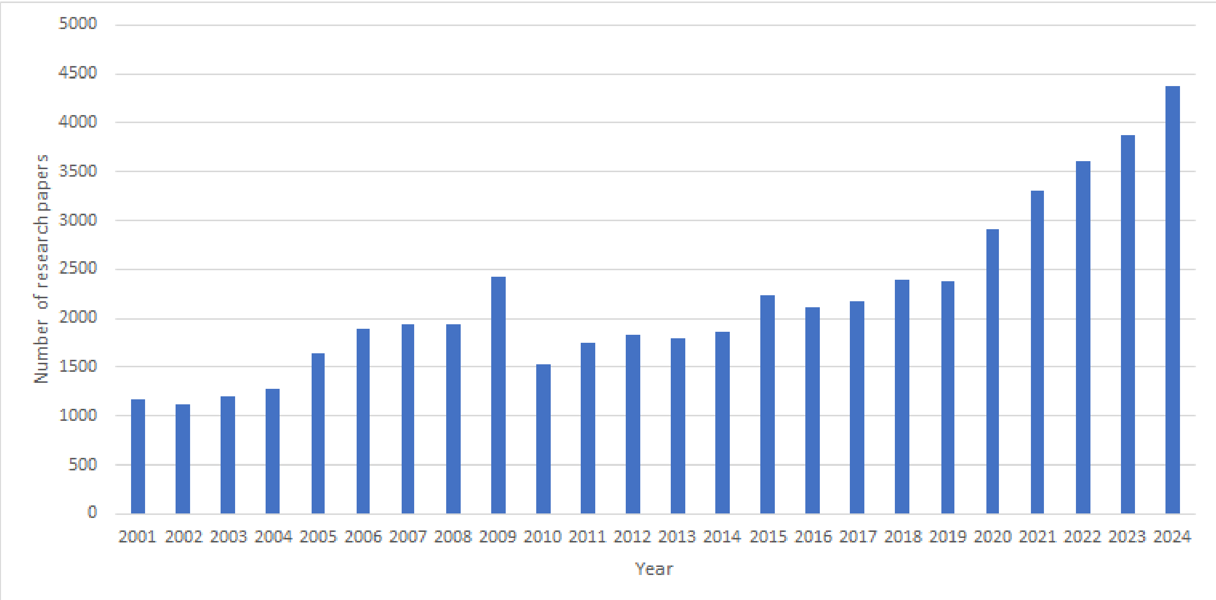

Figure 2. The evolution of research paper number between 2001 and 2024.
Source: Own elaboration using data from www.sciencedirect.com

The ideas of chaos theory began to be taken up in economics in the late 1970s and early 1980s through several pioneering papers (Medio,1979), (Benhabib et al., 1981), (Hsiesh 1991), (Brock & Hsieh, 1991), (Brock et all., 1991). If until then the idea that markets are dynamically unstable played a minor role in the study of economic phenomena, and stochastic models explain instability as a result of external shocks, chaos theory considers that it is the direct result of the dynamics of the system and is determined by the complex interactions between the market's component elements (Gilmore, 1996), (Gilmore, 2001).

The number of research papers concerning chaos theory is continuously growing as one can observe in Figure no. 2. The data were collected querying ScienceDirect database.

3.1. EMBEDDING DIMENSION

An important issue, which is common to all reconstruction methods, is that of determining the minimum size of inclusion. This minimum size is unknown and needs to be determined.

If the inclusion size is incorrectly determined (it is smaller than the optimal value), the attractor will not be fully unfolded i.e. the points located at a distance d in the original phase space will be located at a smaller distance d' in the reconstructed phase space.

It has been shown, according to Sauer's theorem, that the inclusion dimension must respect inequality:

$$m \geq 2d_a + 1 \quad (2)$$

Takens's theorem guarantees that an attractor (d_a – attractor's dimension) embedded in an m-dimensional state space with m chosen according to the above inequality is completely unfolded without self-intersecting. The attractor reconstructed by Takens' method will have the same topological characteristics and geometric shape as the original attractor.

3.2. SENSITIVITY ON INITIAL CONDITIONS

The main feature of a chaotic system is the sensitivity on initial conditions, meaning that the slightest change of the initial conditions can change the long-term behavior of the system.

Sensitive dependence on initial conditions implies that the system's behavior is predictable for only small amounts of time. The horizon of predictability has been proven to depend on the Lyapunov exponent, but recently, some authors found out that an accurate enough prediction could extend beyond the horizon (Martinez, 2018) Authors have found out that it is possible to predict the values of three time series studied (Great Salt Lake elevation, Lorenz and the average number of pieces waiting in line time series) beyond the theoretical barrier imposed by the maximum prediction time.

The Lyapunov exponent measures the speed at which neighboring trajectories diverge. A chaotic system has at least one positive Lyapunov exponent.

The predictability time for a system affected by a disturbance ε with an error Δ is given by:

$$T = \frac{1}{\lambda_{max}} ln \frac{\Delta}{\varepsilon} \quad (3)$$

Where λ_{max} is the maximum Lyapunov exponent.

A chaotic system passes through areas or points in the phase space called chaotic attractors by not repeating the same trajectory. A chaotic system can have in the state phase one or more chaotic attractors towards the system evolves.

3.3. STRANGE ATRACTORS

An attractor is a geometric structure formed by the asymptotic states of the chaotic system, or, in other words, the attractor is formed by a subset of the phase space which attracts all trajectories in this space. The dynamics of the attractor is dictated by contractions and expansions. Expansions occur due to the divergence of nearby trajectories, and compressions due to localized dynamics in the bounded area including the attractor.

A strange attractor will attract the trajectories of the dynamical system, and they will never intersect nor repeat exactly the same paths in phase space. A chaotic attractor is a fractal object due to the density of the trajectories, the details of which will be visible by magnifying

Figure 3 shows Gumowski – Mira attractor described by:

$$x_{n+1} = y_n + a(1 - by_n^2)y_n + Gx(n) \quad (4)$$

$$y_{n+1} = -x_n + G(x_{n+1}) \quad (5)$$

$$G(x) = \mu x + \frac{2(1-\mu)x^2}{1+x^2} \quad (6)$$

Depending on the values of the parameters a, b, μ, fractal phase space representations can be obtained.

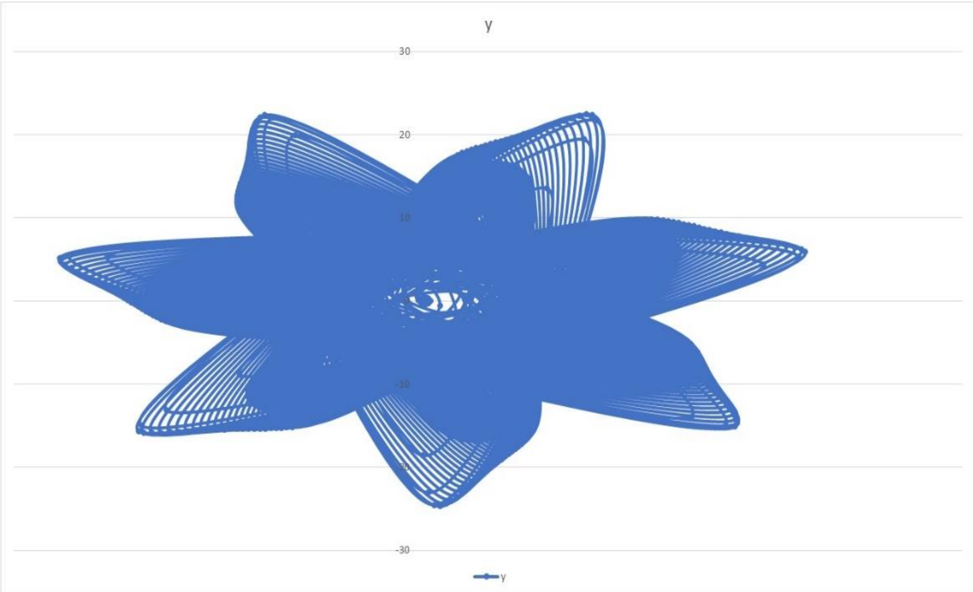

Figure 3. The Gumowski – Mira attractor.
Source: Own elaboration using Microsoft Excel.

4. CONCLUSIONS

Chaos theory triggered a general enthusiasm among scholars. The applications of chaos theory are diverse and could explain the intricate behaviors observed in different research fields that couldn’t be explained otherwise. In economics chaos theory has multiple applications including the attempt to explain the theory of the business cycles using chaos theory (Kantz, 2004). Differences among the findings of the authors approaching the same field were important. For instance, the investigation of chaos occurrence in exchange rate led to different conclusions (Medio et al., 1998), and others identify chaos in exchange rate whilst Brooks doesn’t find any indicators of chaos manifestation (Federici et al., 2002).

The economic system is very complex having many degrees of freedom The subsystems forming the economic system may exhibit chaotic dynamics and therefore interactions produced by these parts of the overall systems can determine the occurrence, at different scales, of a chaotic dynamics. Taking into account that chaos may occur theoretically in every complex nonlinear system and even in much simpler systems, we can ask the question whether chaos is a common feature of any sound economic system.

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