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GENERALIZATION OF THE PRODUCTION FUNCTION OF COBB-DOUGLAS TYPE

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Abstract:
The purpose of this paper is to generalize the operation of Cobb-Douglas type production, thus achieving the interdependence between the resources used and the manufacturing factors in the production process. To achieve the proposed goal, we have independent variables as input variables and variables dependent on this output variable. The input variables are the resources used and the factors of production, and the output variable is the production function. The established results consist in generalizing the Cobb-Douglas-type production function, determining the dependent variable elasticities in relation to each independent variable and marginal productivity, going through the steps of the least squares method and obtaining the coefficients of the Cobb-Douglas-type production function. These analyzes are essential in large econometrics and are useful for researchers who want to use more influencing factors (inputs) in econometric analysis.

Key words: Cobb-Douglas type production function, production factors, elasticity, marginal productivity

JEL classification: C13, C55, O40

1. INTRODUCTION

Economic phenomena are very complex and contain factors of economic growth. First date P.H. Douglas gives a speech on the existence of the theory of production. "It summarized approximately 30 econometric studies that inductively investigated marginal productivity theory. It was this body of work that yielded the Cobb-Douglas production function that is used extensively in theoretical and applied research. Douglas undertook this research because he had reached the conclusion that economic theorists had become lazy in how they illustrated marginal productivity curves" (Fraser, 2002). Cobb and Douglas studies, although challenged, have led to the formulation of marginal productivity theory.

With the help of the Cobb-Douglas production function we can analyze the influence of the resources used and the production factors on the achievable production level.

The Cobb-Douglas-type production function is used in micro and macroeconomics and represents the interdependence between the resources used in the production process and the level of production that can be achieved with the help of the technologies used. It expresses the connections between inputs and outputs in production structures, indicating the maximum value of production. Supply theory is based on the function of production. In the decision process, production decisions have an essential role by managing inputs (resources used and production factors) and outputs (production function). "The decision involves the combination or substitution of production factors and involves changing the amount, of factors used, changing the proportion between factors or replacing one factor with another. The substitution of production factors has been and is a field of research in economics, given the trend of dematerialization of production through the development of information and communication technology, ICT, and increasing the importance of services in the knowledge-based society" (Iacob and Dumitru, 2020). For this reason, in this paper we aim to generalize the Cobb-Douglas-type production function by considering the involvement in the production process of several resources used and factors of production (several inputs). The Cobb-Douglas-type production function "without restrictions further increases its potential to manage different production scales" (Bhanumurthy, 2002).

The manager has an essential role in the decision-making act, being concerned with the efficiency of the resources used and the production factors. In this context, the purpose of the research is to generalize the function of Cobb-Douglas production, thus achieving the interdependence between the resources used and the manufacturing factors in the production process. In order to achieve the proposed goal, the following objectives are established: 1. Generalization of the Cobb-Douglas-type production function and 2. Determination of the regression coefficients for the Cobb-Douglas-type production function. The results obtained are essential in large econometric analyzes and are useful for researchers who want to use in econometric analysis, several influencing factors (inputs). In this article, the influencing factors are the resources used and the production factors, but social or time factors can be included, because "globalization, the creation of the single market for goods and services in the European Union and the fluidization of capital flows have led to essential fiscal policy changes." (Tulvinschi and Macovei).

The aim of the research is to generalize the function of Cobb-Douglas-type production, thus achieving the interdependence between the resources used and the manufacturing factors in the production process. In order to achieve the proposed goal, the following objectives are set:

O1. Generalization of the Cobb-Douglas-type production function;

O2. Determination of regression coefficients for the Cobb-Douglas-type production function.

In order to achieve the proposed goal, specific methodological instruments will be used, namely techniques specific to the researched field.

2. SPECIALTY LITERATURE

The production function (Cobb-Douglas) is researched and applied by specialists in the field to study economic growth, a factor of economic sustainability. In the literature we find numerous works that have as subjects of study the Cobb-Douglas type production function and the economic modeling performed using this function. Due to the increasingly complex economic phenomena, many leading specialists in the field use the Cobb-Douglas type production function in econometric analyzes on their evolution.

The Cobb-Douglas-type production function used and estimated by Cobb and Douglas in the 1928 paper and other subsequent works, as well as by many researchers in the field, has the form:

$$Q = AL^{\alpha_1} K^{\alpha_2}$$

where Q represents production, L represents labor, K represents capital, and A, α_1 and α_2 are constant. The authors assume that $\alpha_1 + \alpha_2 = 1$ and the conditions of collinearity are met.

In the paper (Chen and Han, 2014) the authors determine a new study model of the Cobb-Douglas-type production function to calculate the rate of contribution of various influencing factors to economic growth in China and propose a new algorithm that is characterized by a "high precision and fast convergence". The indicators of the efficiency of the economic potential were studied for the Cobb-Douglas type production function (Anghel and all, 2019). The indicators of invested capital and labor force influence production (Macovei and Siretean, 2017). The Cobb-Douglas type production function is used to influence the performance of the supply on the measurement of the productive potential at microeconomic level. Due to the increase in the workforce in the paper (Hájková and Hurník, 2007), the authors test whether the production function can be applied to the Czech economy. Indicators for assessing efficiency and productivity are modeled with the Cobb-Douglas-type production function (Stan, 2005).

The authors of the paper (Zellner and all, 1966) review the "traditional" hypotheses specific to the Cobb-Douglas-type production function model that maximizes profit and develops "a model in which profits are stochastic and in which maximizing the mathematical expectation of profits."

In the paper (Macovei and Balan, 2009) the indicators the number of tourism employees, the number of accommodation units and the number of accommodated tourists influence the turnover according to the Cobb-Douglas type production function. The author of the paper (Bhanumurthy, 2002) argues that the Cobb-Douglas production function "deserves to be used for analysis of the production process, not because it should be seen as a simple tool that can be easily manipulated or as a raw remedy for estimation, but from because of the advantages it offers (it possesses). These advantages are due to the fact that it can manage multiple entries in its generalized form. Even in the face of market imperfections, it does not introduce its own distortions. "

Variable elasticity of Cobb-Douglas-type production functions are treated and calculated in the works (Lu, 1967) and (Gechert and all, 2019).

3. DEFINITION OF THE GENERALIZED PRODUCTION FUNCTION

We consider the generalization of the Cobb-Douglas type production function of the form:

$$Y = \beta \prod_{i=1}^{m} X_i^{\alpha_i} \text{, cu } \alpha_i \in [-1,1], i = \overline{1,m}, \beta > 0 . \tag{1}$$

where:

- Y represents the level of production and is a form-dependent variable $Y = (y_1, y_2, \dots, y_n)$, $y_j > 0,\ j = \overline{1,n}$;
- $X_i,\ i = \overline{1,m}$ represent the resources used and the factors of production in the production process and are form-independent variables $X_i = (x_{i1}, x_{i2}, \dots, x_{in})$; $x_{ij} > 0, i = \overline{1,m},\ j = \overline{1,n}$;
- β represents the regression coefficient that shows the mean value of the dependent variable Y when $X_i = 1,\ i = \overline{1,m}$;
- $\alpha_i,\ i = \overline{1,m}$ represents the regression coefficients, which are the elasticities of the dependent variable in relation to each independent variable, where

$$\alpha_i = \frac{X_i}{Y} \frac{\partial Y}{\partial X_i},\ i = \overline{1,m} .$$

It has been shown over time that production processes follow a "homogeneous linear function with a substitution elasticity of one of the factors" (Douglas, 1976). The regression coefficients must meet the collinearity properties.

Using the production function, the managers aim in the production process to maximize the production using the resources and technologies. Marginal productivity is given by the formula:

$$\frac{\partial Y}{\partial X_i} > 0 , i = \overline{1,m}$$

and represents the additional production obtained due to the additional use of a unit of the resource used under the conditions in which the other resources in the production process remain constant. Marginal productivity is evolving as well is decreasing. The evolution of the marginal productivity curve is based on the law of yield, namely there is a point where marginal productivity decreases when one resource increases and the other resources remain constant.

4. DETERMINATION OF THE COEFFICIENTS OF THE PRODUCTION FUNCTION

In estimating the coefficients of the Cobb-Douglas-type production function, the estimator to be used depends on the specification of the behavior of the disturbance term in the production function. If the disturbance term is not transmitted to the inputs, if the inputs are independent of this disturbance, then the least squares estimator is consistent, and if the perturbation is transmitted completely to the inputs, then a consistent estimator is obtained if some restrictions are imposed on the input. the second moment of system disturbances. However, a more general case may be specified, including the above specifications as sub-cases. In this general case, the disturbance term can only be transmitted partially. If this happens, then none of the estimators mentioned above are consistent. In fairly general situations, these estimators provide upper and lower limits for the elasticity of the production function (in a case with a single input) or for the sum of the elasticities. The consequences of each of these specifications, in terms of probability limits, are examined in detail. This is done first, for an input case, then the input case is discussed.

The Cobb-Douglas-type production function is a multiple nonlinear regression function. To determine the regression coefficients, we transform the Cobb-Douglas production function (1) into a multiple log-linear model of the form:

$$\ln Y = \ln\beta + \sum_{i=1}^{m}\alpha_i \ln X_i + \varepsilon \text{ , cu } \alpha_i \in [-1,1],\ i=\overline{1,m},\ \beta > 0 .$$

Under these conditions we have the shape error function:

$$\varepsilon = \ln Y - \ln\beta - \sum_{i=1}^{m}\alpha_i \ln X_i \text{ , } \alpha_i \in [-1,1],\ i=\overline{1,m},\ \beta > 0$$

We customize the quadratic error function for a sample *n*:

$$S(\ln\beta,\alpha_1,\alpha_2,\ldots,\alpha_m) = \sum_{j=1}^{n}\left(\ln y_j - \ln\beta - \sum_{i=1}^{m}\alpha_i \ln x_{ij}\right)^2 \text{ , } \alpha_i \in [-1,1],\ i=\overline{1,m},\ \beta > 0$$

and we have an extremely local problem. For ease of calculation we note. That's how we have the function:

$$S(b,\alpha_1,\alpha_2,\ldots,\alpha_m) = \sum_{j=1}^{n}\left(\ln y_j - b - \sum_{i=1}^{m}\alpha_i \ln x_{ij}\right)^2$$

If we derive the function $S(b,\alpha_1,\alpha_2,\ldots,\alpha_m)$ in relation to the variables $b,\alpha_1,\alpha_2,\ldots,\alpha_m$ then we get:

$$\frac{\partial S}{\partial b} = -2\sum_{j=1}^{n}\left(\ln y_j - b - \sum_{i=1}^{m}\alpha_i \ln x_{ij}\right)$$
$$\frac{\partial S}{\partial \alpha_1} = 2\sum_{j=1}^{n}\left(\ln y_j - b - \sum_{i=1}^{m}\alpha_i \ln x_{ij}\right)\left(-\ln x_{1j}\right)$$

$$\frac{\partial S}{\partial \alpha_2} = 2\sum_{j=1}^{n}\left(\ln y_j - b - \sum_{i=1}^{m}\alpha_i \ln x_{ij} \right)\left(-\ln x_{2j}\right)$$
$$\vdots$$
$$\frac{\partial S}{\partial \alpha_m} = 2\sum_{j=1}^{n}\left(\ln y_j - b - \sum_{i=1}^{m}\alpha_i \ln x_{ij} \right)\left(-\ln x_{mj}\right)$$

According to the algorithm for determining the local extremes we have the equations, which lead to the normal Gaussian system:

$$\begin{cases} \sum_{j=1}^{n}\left(\ln y_j - b - \sum_{i=1}^{m}\alpha_i \ln x_{ij} \right) = 0 \\ \sum_{j=1}^{n}\left(\ln y_j - b - \sum_{i=1}^{m}\alpha_i \ln x_{ij} \right)\left(-\ln x_{1j}\right) = 0 \\ \sum_{j=1}^{n}\left(\ln y_j - b - \sum_{i=1}^{m}\alpha_i \ln x_{ij} \right)\left(-\ln x_{2j}\right) = 0 \\ \vdots \\ \sum_{j=1}^{n}\left(\ln y_j - b - \sum_{i=1}^{m}\alpha_i \ln x_{ij} \right)\left(-\ln x_{mj}\right) = 0 \end{cases}$$

Performing the calculations we get the system:

$$\begin{cases} n\,b + \sum_{i=1}^{m}\alpha_i \ln x_{ij} = \sum_{j=1}^{n}\ln y_j \\ b\sum_{j=1}^{n}\ln x_{1j} + \sum_{j=1}^{n}\left(\sum_{i=1}^{m}\alpha_i \ln x_{ij} \right)\ln x_{1j} = \sum_{j=1}^{n}\left(\ln y_j \ln x_{1j}\right) \\ b\sum_{j=1}^{n}\ln x_{2j} + \sum_{j=1}^{n}\left(\sum_{i=1}^{m}\alpha_i \ln x_{ij} \right)\ln x_{2j} = \sum_{j=1}^{n}\left(\ln y_j \ln x_{2j}\right) \\ \vdots \\ b\sum_{j=1}^{n}\ln x_{2j} + \sum_{j=1}^{n}\left(\sum_{i=1}^{m}\alpha_i \ln x_{ij} \right)\ln x_{mj} = \sum_{j=1}^{n}\left(\ln y_j \ln x_{mj}\right) \end{cases} \Rightarrow \begin{cases} b = \hat{b} \\ \alpha_1 = \hat{\alpha}_1 \\ \alpha_2 = \hat{\alpha}_2 \\ \vdots \\ \alpha_m = \hat{\alpha}_m \end{cases} \Rightarrow \begin{cases} \beta = e^{\hat{b}} \\ \alpha_1 = \hat{\alpha}_1 \\ \alpha_2 = \hat{\alpha}_2 \\ \vdots \\ \alpha_m = \hat{\alpha}_m \end{cases}$$

So, we get the unique solution $(\hat{b},\ \hat{\alpha}_1,\ \hat{\alpha}_2,\ \ldots,\ \hat{\alpha}_m)$. Calculated elasticities $\hat{\alpha}_i$, $i = \overline{1,m}$ represents the percentage increase of the production function (outputs) in relation to the percentage increase of the resources used and the factors production (inputs).

Calculating second-order partial derivatives for the function $S(b,\alpha_1,\alpha_2,\ldots,\alpha_m)$, get:

$$\frac{\partial^2 S}{\partial b^2} = 2n,\ \frac{\partial^2 S}{\partial b\,\partial \alpha_i} = 2\sum_{j=1}^{n}\ln x_{ij},\ \frac{\partial^2 S}{\partial \alpha_i^{\,2}} = 2\sum_{j=1}^{n}\left(\ln x_{ij}\right)^2,$$
$$\frac{\partial^2 S}{\partial \alpha_i\,\partial \alpha_k} = 2\sum_{j=1}^{n}\ln x_{ij}\ln x_{kj},\ i = \overline{1,m},\ k = \overline{1,m},\ i \neq k.$$

The Hessian matrix has the shape:

$$H(b,\alpha_1,\alpha_2,\dots,\alpha_m)=\begin{bmatrix} 2n & 2\sum_{j=1}^{n}\ln x_{1j} & 2\sum_{j=1}^{n}\ln x_{2j} & \cdots & 2\sum_{j=1}^{n}\ln x_{mj} \\ 2\sum_{j=1}^{n}\ln x_{1j} & 2\sum_{j=1}^{n}\left(\ln x_{1j}\right)^2 & 2\sum_{j=1}^{n}\ln x_{1j}\,\ln x_{2j} & \cdots & 2\sum_{j=1}^{n}\ln x_{1j}\,\ln x_{mj} \\ 2\sum_{j=1}^{n}\ln x_{2j} & 2\sum_{j=1}^{n}\ln x_{1j}\,\ln x_{2j} & 2\sum_{j=1}^{n}\left(\ln x_{2j}\right)^2 & \cdots & 2\sum_{j=1}^{n}\ln x_{2j}\,\ln x_{mj} \\ \vdots & \vdots & \vdots & & \vdots \\ 2\sum_{j=1}^{n}\ln x_{mj} & 2\sum_{j=1}^{n}\ln x_{1j}\,\ln x_{mj} & 2\sum_{j=1}^{n}\ln x_{2j}\,\ln x_{mj} & \cdots & 2\sum_{j=1}^{n}\left(\ln x_{mj}\right)^2 \end{bmatrix}$$

According to the algorithm for determining the local extremes we have:

$$\Delta_1 = 2n > 0$$

$$\Delta_2 = \begin{vmatrix} 2n & 2\sum_{j=1}^{n}\ln x_{1j} \\ 2\sum_{j=1}^{n}\ln x_{1j} & 2\sum_{j=1}^{n}\left(\ln x_{1j}\right)^2 \end{vmatrix} > 0$$

$$\Delta_3 = \begin{vmatrix} 2n & 2\sum_{j=1}^{n}\ln x_{1j} & 2\sum_{j=1}^{n}\ln x_{2j} \\ 2\sum_{j=1}^{n}\ln x_{1j} & 2\sum_{j=1}^{n}\left(\ln x_{1j}\right)^2 & 2\sum_{j=1}^{n}\ln x_{1j}\,\ln x_{2j} \\ 2\sum_{j=1}^{n}\ln x_{2j} & 2\sum_{j=1}^{n}\ln x_{1j}\,\ln x_{2j} & 2\sum_{j=1}^{n}\left(\ln x_{2j}\right)^2 \end{vmatrix} > 0$$

$$\vdots$$

$$\Delta_{m+1} = \begin{vmatrix} 2n & 2\sum_{j=1}^{n}\ln x_{1j} & 2\sum_{j=1}^{n}\ln x_{2j} & \cdots & 2\sum_{j=1}^{n}\ln x_{mj} \\ 2\sum_{j=1}^{n}\ln x_{1j} & 2\sum_{j=1}^{n}\left(\ln x_{1j}\right)^2 & 2\sum_{j=1}^{n}\ln x_{1j}\,\ln x_{2j} & \cdots & 2\sum_{j=1}^{n}\ln x_{1j}\,\ln x_{mj} \\ 2\sum_{j=1}^{n}\ln x_{2j} & 2\sum_{j=1}^{n}\ln x_{1j}\,\ln x_{2j} & 2\sum_{j=1}^{n}\left(\ln x_{2j}\right)^2 & \cdots & 2\sum_{j=1}^{n}\ln x_{2j}\,\ln x_{mj} \\ \vdots & \vdots & \vdots & & \vdots \\ 2\sum_{j=1}^{n}\ln x_{mj} & 2\sum_{j=1}^{n}\ln x_{1j}\,\ln x_{mj} & 2\sum_{j=1}^{n}\ln x_{2j}\,\ln x_{mj} & \cdots & 2\sum_{j=1}^{n}\left(\ln x_{mj}\right)^2 \end{vmatrix} > 0$$

Under these conditions, the solution $(\hat{b},\ \hat{\alpha}_1,\ \hat{\alpha}_2,\ \ldots,\hat{\alpha}_m)$ it is minimum local point, so it minimizes the square deviation.

Total elasticity is given by the relationship:

$$\alpha = \hat{\alpha}_1 + \hat{\alpha}_2 + \ldots + \hat{\alpha}_m$$

and measures the percentage increase in the level of production when the resources used and the factors of production increase in the same proportions. In these conditions we have:

- If $\hat{\alpha}_1 + \hat{\alpha}_2 + \ldots + \hat{\alpha}_m > 1$, then the variation of production is accelerated and is encountered when the production function (outputs) increases in a greater proportion than the resources used and the production factors (inputs) increase;
- If $\hat{\alpha}_1 + \hat{\alpha}_2 + \ldots + \hat{\alpha}_m = 1$, then the variation of production is constant and occurs when the production function (outputs) increases in the same proportion as the increase of resources used and factors of production (inputs);
- If $\hat{\alpha}_1 + \hat{\alpha}_2 + \ldots + \hat{\alpha}_m < 1$, then the variation of production is low and is found when the production function (outputs) increases in a smaller proportion than the resources used and the production factors (inputs) increase.

The generalization of the Cobb-Douglas-type production function achieves the interdependence between the resources used and the manufacturing factors in the production process. Thus this generalized function for which we have a large number of variables can be used in works in various more complex fields.

5. CONCLUSION

The Cobb-Douglas-type production function occupies an essential place in the econometric analysis of national and international economic phenomena. This function serves the manager to understand the fundamental mechanisms of the decision act from a theoretical point of view, but also from a practical point of view. Due to the increasingly complex economic phenomena, many leading specialists in the field use the Cobb-Douglas type production function in econometric analysis to predict their evolution.

A generalization of the Cobb-Douglas-type production function, which is a nonlinear regression model, has an essential role in the decision act. In the paper, the objectives were achieved, namely a Cobb-Douglas-type production function was generalized and the regression coefficients for the generalized function were determined. Therefore, the purpose of the research is achieved.

The results obtained are essential in increasingly complex econometric analyzes being useful to researchers who want in econometric analysis to use more independent variables, resources used and production factors but may include social or time factors (inputs).

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