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ECONOMIC GROWTH AND WEALTH ACCUMULATION WITH PERFECT AND MONOPOLISTIC COMPETITION

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Abstract:
This study constructs a growth model by integrating the three basic models in neoclassical growth theory with perfect competition and general equilibrium theory with monopolistic competition. The economy is composed of three sectors – the final good sector as in Solow's one sector growth model, the middle goods sector for final good as Grossman and Helpman's intermediates sector (which supplies intermediate inputs for the final good sector), and the middle goods sector for consumption as Dixit and Stiglitz's intermediates sector (which supplies goods for consumption). Our model is different from the Solow model in that we model the household behavior with the utility function and disposable income proposed by Zhang. We deviate from the Dixit-Stiglitz model and the Grossman- Helpman model in that we distribute the profits of firms in monopolistic competition to households. We build and then simulate the model. We demonstrate the existence of a unique stable equilibrium point and plot the motion of the economy. We examine the effects of exogenous changes in some parameters. The comparative dynamic analysis analyzes the effects of the exogenous changes on transitory process and long-term equilibrium structure.

Key words: Solow model; perfect competition; Dixit-Stiglitz model; Grossman and Helpman model; monopolistic competition; wealth accumulation; profit distribution; general economic equilibrium.

JEL classification: O41

1. INTRODUCTION

Many modern economies are characterized by coexistence of perfectly and imperfectly competitive industries. There are many approaches in macroeconomics in explaining national economic structures with different markets. Different from traditional economic theory which mainly deals with perfectly competitive markets, new economic theory is more concerned with monopolistic competition. As real world is characterized by so many types of markets and different types of games among firms, it is easy to see limitations of perfect competition approach and limitations of monopolistic competition approach. The purpose of this study is to integrate two important modelling frameworks – neoclassical growth theory with perfect competition and new economic theory with monopolistic competition. As the literature in each approach is huge and very complicated, we are concerned with the basic models in each approach.

Theory of monopolistic competition developed in recent years is concerned with vast issues related to industrial structure and endogenous growth with monopolistic competition. In their work on monopolistic competition and optimum product diversity, Dixit and Stiglitz (1977) emphasize great diversity of consumption in which intermediate goods are used as consumer goods. Different from Dixit and Stiglitz, Grossman and Helpman (1990) study an economy with a wide array of differentiated intermediate inputs which are used by the final goods sector. Partly because their models are structurally simple and analytically tractable, they have got much attention among economists who are interested in modelling market structures with monopolistic competition. This study integrates the two models with the Solow one-sector growth model with perfect competition. It should be noted that Chamberlin (1933) makes the seminal contribution to formal theory of monopolistic competition. The theory of monopolistic competition has recently been applied in different fields of economics in modelling modern economic structures, economic growth and development, economic geography, and innovation and technological diffusion (e.g., Lancaster, 1980; Waterson, 1984; Benassy, 1996; Bertoletti and Etro, 2015; Nocco, *et. al*., 2017; and Parenti, *et.al*., 2017). The modelling framework by Dixit and Stiglitz (1977) has caused special attention as it provides an analytically tractable modelling framework. It makes an important impact on the development of the literature of formally modelling monopolistic competition (e.g., Krugman, 1979; Ethier, 1982; Romer, 1990; Brakman and Heijdra, 2004; Behrens and Murata, 2007, 2009; Wang, 2012). Zhang (2018) contributes the literature by introducing monopolistic competition to neoclassical growth theory by applying the modelling strategy by Dixit and Stiglitz (1977) and the literature of economic equilibrium with monopolistic competition. The study further generalizes Zhang's model by introducing consumer variety according to Grossman and Helpman to neoclassical growth theory.

Wealth accumulation is a key machine of economic growth. Nevertheless, new economic theory with monopolistic competition fails to provide an effective modelling framework to introduce endogenous wealth accumulation. On the other hand, neoclassical growth theory is mainly concerned with wealth accumulation with perfect competition. Wealth accumulation is the main growth machine in neoclassical growth theory. Rather than following the mainstream neoclassical growth theory with the Ramsey approach, Zhang (2005) introduced an alternative utility and disposable income to neoclassical growth theory. Zhang's modelling framework makes it possible to analyze many important economic problems. Although this study follows Zhang's approach to household behavior, we model production of final good and perfect competition by following traditional neoclassical growth theory. The seminal work for development of neoclassical growth theory is due to Solow (1956). Solow's one sector growth model shows simply and logically how the economic growth rate is determined with exogenous saving, exogenous technological change, and exogenous population growth in a perfectly competitive economy. Over years strict assumptions in the Solow model has been relaxed in different directions (e.g., Burmeister and Dobell, 1970; Azariadis, 1993; Barro and Sala-i-Martin, 1995; Ben-David and Loewy, 2003, and Zhang, 2005, 2008). This study makes a further extension of neoclassical growth model by integrating the Solow model with the two basic models in the literature of monopolistic competition.

This study integrates the three basic models in neoclassical growth theory with perfect competition and general equilibrium theory with monopolistic competition. We differ from the Dixit-Stiglitz model and Grossman-Helpman model in that the profits of intermediate inputs sectors are distributed among the homogeneous population rather than used for investment. The rest of the paper is organized as follows. Section 2 constructs the three-sector growth model with perfect competition and monopolistic competition. Section 3 examines the model and illustrate properties of the economic system by simulation. Section 4 conducts comparative dynamic analysis in some parameters. Section 5 concludes the study.

2. THE NEOCLASSICAL GROWTH MODEL WITH INTERMEDIATE INPUTS

We synthesize the three important models to show equilibrium with perfect competition and monopolistic competition. The three models are the Solow one-sector neoclassical growth model, the Grossman-Helpman model with consumer variety, and the Dixit-Stiglitz model with product variety. These models are integrated with applying Zhang's concept of disposable income and utility function. With regard to modelling production, we follow the analytical framework for monopolistic competition by Dixit and Stiglitz (1977) and Grossman and Helpman (1990). Monopolistic competition is characterized by many producers who produce differentiated products. Products are differentiated from each other and are not perfect substitutes. Each firm takes the prices charged by other firms as given and maximizes its profit. Each firm has some degree of market power. Market power is measured by power over the terms and conditions of demand and supply equilibrium. The model in this study is to extend Zhang's model (Zhang, 2018) by introducing consumer variety by Dixit and Stiglitz (1977). The supply side consists of three kinds of activities: production of a final (capital) good, the production of a variety of differentiated middle products (i.e., intermediate inputs) for final good sector, and the production of a variety of differentiated middle products for consumption. No firm in the middle goods sectors can produce a product with an attribute that is very close to any given attribute of any other product. They are assumed to act atomistically in that no firm take account of possible impacts of its decisions on any other firm. Capital good is the same as the commodity in the Solow model, which can be invested as capital good and consumed as consumer good.

Final good sector

The final good sector is similar to the industrial sector in the Solow model, except that intemediates are used as inputs. Let $F_i(t)$, $K(t)$ and $N_i(t)$ stand for, respectively, output of the final goods sector, capital input and labor input. As in Dixit and Stiglitz (1977) and Grossman and Helpman (1990), we use $X_i(t)$ to represent aggregate input of intermediate inputs as follows:

$$X_i(t) = \sum_{\varepsilon=1}^{n} x_\varepsilon^\theta(t)\,, 0 < \theta < 1, \qquad (1)$$

in which $x_\varepsilon(t)$ represents the input of middle product ε, n is the number of varieties of middle products available, and θ is a parameter. As Grossman and Helpman (1990), the production function of final goods is specified as:

$$F_i(t) = A_i K^{\alpha_i}(t) N_i^{\beta_i}(t) X_i^{\gamma_i}(t)\,, \qquad 0 < \alpha_i, \beta_i, \alpha_i + \beta_i < 1, \qquad \gamma_i = \frac{1 - \alpha_i - \beta_i}{\theta} < 1,$$

in which A_i, α_i and β_i are coefficients. The production function is constant returns to scale for given n, but exhibits an increase in n. The specification has the property that an increasing degree of specialization enhances technical efficiency. Here, a rise in n implies increasing the degree of specialization. There are scale economies at the industry level. The scale economies are exogenous to the individual firms in the final goods sector.

We use capital good serves as a medium of exchange. It is taken as numeraire. Physical capital depreciates at a constant exponential rate δ_k. We use $w(t)$, $r(t)$, and $p_\varepsilon(t)$, to stand for, respectively, the wage rate, the rate of interest, and the price of middle good ε. We have the profit of the capital good sector as follows:

$$\pi_i(t) = F_i(t) - (r(t) + \delta_k)K(t) - w(t)N_i(t) - \sum_{\varepsilon=1}^{n} p_\varepsilon(t) x_\varepsilon(t). \quad (2)$$

Maximizing profit implies the following marginal conditions:

$$r(t) + \delta_k = \frac{\alpha_i F_i(t)}{K(t)}, \qquad w(t) = \frac{\beta_i F_i(t)}{N_i(t)}, \qquad p_\varepsilon(t) = \frac{\gamma_i \theta x_\varepsilon^{\theta-1}(t) F_i(t)}{X_i(t)}. \quad (3)$$

The production function implies that the share of factor X_i is $\gamma_i F_i$. From (2) and (3) we have the following relations:

$$K(t) = \Lambda(t) X_i(t), \qquad N_i(t) = \left(\frac{w(t)}{\beta_i A_i K^{\alpha_i}(t) X_i^{\gamma_i}(t)}\right)^{1/(\beta_i - 1)}, \quad (4)$$

where

$$\Lambda(r, w, t) \equiv \left[\left(\frac{r_\delta(t)}{\alpha_i\, A_i}\right)^{\beta_i - 1} \left(\frac{\beta_i\, A_i}{w(t)}\right)^{\beta_i}\right]^{\frac{1}{\gamma_i}}, \qquad r_\delta(t) \equiv r(t) + \delta_k.$$

We see $\Lambda(t)$ independent of variety. From (3), we get

$$p_\varepsilon(t) = \frac{\gamma_i \, \theta \, r_\delta(t) \, x_\varepsilon^{\theta-1}(t) \, K(t)}{\alpha_i \, X_i(t)}. \quad (5)$$

Inserting (4) in (5) implies:

$$x_\varepsilon(t) = \tilde{\Lambda}(t) p_\varepsilon^{-\tilde{\theta}}(t), \quad (6)$$

in which

$$\tilde{\Lambda}(t) \equiv \left(\frac{\gamma_i \theta_i r_\delta(t) \Lambda(t)}{\alpha_i}\right)^{\tilde{\theta}}, \quad \tilde{\theta} \equiv \frac{1}{1-\theta}.$$

We see that $\tilde{\Lambda}(t)$ is independent of variety. The share of variety ε in the total value of intermediate inputs is

$$\varphi_\varepsilon(t) \equiv \frac{x_\varepsilon(t) p_\varepsilon(t)}{\sum_{m=1}^{n} x_m(t) p_m(t)}. \quad (7)$$

Insert (6) in (7)

$$\varphi_\varepsilon(t) = \frac{p_\varepsilon^{1-\tilde{\theta}}(t)}{\sum_{m=1}^{n} p_m^{1-\tilde{\theta}}(t)}. \quad (8)$$

The middle goods sector for final good

Following Dixit and Stiglitz (1977) and Grossman and Helpman (1990), we describe production of middle goods for final good. The profit is the product of profits per unit of product and the share of the market. The profit of the producer of variety ε is as follows:

$$\pi_\varepsilon(t) = [p_\varepsilon(t) - a_N \, w(t)] \frac{\varphi_\varepsilon(t) \, \gamma_i \, F_i(t)}{p_\varepsilon(t)},$$

where a_N is the unit labor requirement for production of intermediates. Substituting (8) into the above profit implies:

$$\pi_\varepsilon(t) = [p_\varepsilon(t) - a_N \, w(t)] \frac{\gamma_i \, F_i(t) \, p_\varepsilon^{-\tilde{\theta}}(t)}{\sum_{m=1}^{n} p_m^{1-\tilde{\theta}}(t)}. \quad (9)$$

The producer chooses $p_\varepsilon(t)$ to maximizes the profit. With (3) and (1), we get

$$F_i(t) = \frac{p_\varepsilon(t)}{\gamma_i \, \theta \, x_\varepsilon^{\theta-1}(t)} \sum_{\varepsilon=1}^{n} x_\varepsilon^{\theta}(t). \quad (10)$$

Substitute (6) into (10)

$$\frac{F_i(t)}{\sum_{m=1}^{n} p_m^{1-\tilde{\theta}}(t)} = \frac{\tilde{\Lambda}(t)}{\gamma_i \theta}. \quad (11)$$

With (9) and (11), we have the profit function as follows:

$$\pi_\varepsilon(t) = [p_\varepsilon(t) - a_N w(t)] \frac{\gamma_i \tilde{\Lambda}(t) p_\varepsilon^{-\tilde{\theta}}(t)}{\gamma_i \theta}.$$

The first-order condition (i.e., $\partial \pi_\varepsilon / \partial p_\varepsilon = 0$) results in the following fixed-markup pricing rule:

$$\theta p_\varepsilon(t) = a_N w(t). \quad (12)$$

By this equation we conclude that varieties bear the same price, denoted by $p_i(t)$. From (9) and (12), we have the profit per firm as follows:

$$\pi_i(t) = \frac{(1-\theta) \gamma_i F_i(t)}{n}, \quad (13)$$

which is independent of ε. From (5), we also conclude that $x_\varepsilon(t)$ is independent of ε, denoted by $x(t)$. From (1) we get

$$X_i(t) = n x^\theta(t). \quad (14)$$

The total profit is

$$\bar{\pi}_i(t) = n \pi_i(t). \quad (15)$$

Consumer behaviors and wealth dynamics

Rather than traditional approaches to household behavior in economic theory, we use an alternative approach to modeling behavior of households. The model is proposed by by Zhang (1993) and has been applied to different fields of economics (Zhang, 2005). We use $\bar{k}(t)$ to represent per household wealth. We have $\bar{k}(t) = K(t)/N$. As in Zhang (2018), we assume that the total profit of all the firms in the two middle good sectors is equally shared among households. We use h to represent human capital of the household and $\pi(t)$ the profit per household. The current income of the representative household is

$$y(t) = r(t) \bar{k}(t) + h w(t) + \pi(t). \quad (13)$$

The household disposable income $\hat{y}(t)$ is the sum of the current disposable income and the value of wealth:

$$\hat{y}(t) = y(t) + \bar{k}(t) = (1 + r(t)) \bar{k}(t) + h w(t) + \pi(t). \quad (14)$$

The representative household spends the total available budget on saving $s(t)$, and consuming final good $c_i(t)$ and intermediates $c_\epsilon(t)$, $\epsilon = n+1, \dots, \bar{n}$. We have the budget constraint as follows;

$$c_i(t) + \sum_{\varepsilon=n+1}^{\bar{n}} p_\varepsilon(t) c_\varepsilon(t) + s(t) = \hat{y}(t). \quad (15)$$

In our model the household decides consumption levels of goods and services and saving. As in Dixit and Stiglitz (1977), we introduce a composite good as follows:

$$c(t) = \left(\sum_{\varepsilon=n+1}^{\bar{n}} c_{\varepsilon}^{\bar{\theta}}(t)\right)^{1/\bar{\theta}}, \quad 1 > \bar{\theta} > 0. \quad (16)$$

We assume that utility level $U(t)$ is dependent on $c_i(t)$, $c(t)$ and $s(t)$ as follows:

$$U(t) = c_i^{\xi_0}(t)\, s^{\lambda_0}(t)\, c^{\chi_0}(t), \quad \xi_0, \lambda_0, \chi_0 > 0, \quad (17)$$

where χ_0 is the propensity to consume intermediates, ξ_0 is the propensity to consume final good, and λ_0 is the propensity to save.

The problem is to maximize utility (17) subject to budget constraint (16). We apply the two-stage method to solve the optimization problem (Dixit-Stiglitz, 1977; Chang, 2012). In the fist stage, we imagine that there is a price $p(t)$ for $c(t)$. The budget for the question is:

$$c_i(t) + p(t)\, c(t) + s(t) = \hat{y}(t).$$

We omit time variables for a while, when there should be no confusion. It is straightforward to show that the optimal solution is given by:

$$c_i = \xi\, \hat{y}, \quad p\, c = \chi\, \hat{y}, \quad s = \lambda\, \hat{y}, \qquad (18)$$

where

$$\xi \equiv \rho\, \xi_0, \quad \chi \equiv \rho\, \chi_0, \quad \lambda \equiv \rho\, \lambda_0, \quad \rho \equiv \frac{1}{\xi_0 + \chi_0 + \lambda_0}.$$

The second-stage maximization is formed as follows: maximize c by choosing c_ϵ subject to:

$$\sum_{\varepsilon=n+1}^{\bar{n}} p_\varepsilon\, c_\varepsilon = p\, c. \quad (19)$$

Introduce the following Langrangian function

$$\mathcal{L} = \left(\sum_{\varepsilon=n+1}^{\bar{n}} c_\varepsilon^{\bar{\theta}}\right)^{1/\bar{\theta}} + \hbar \left\{\chi\, \hat{y} - \sum_{\varepsilon=n+1}^{\bar{n}} p_\varepsilon\, c_\varepsilon\right\}, \quad (19)$$

where $\hbar$ is the Lagrangian multiplier. From (19) we get the following first-order conditions;

$$\frac{\partial \mathcal{L}}{\partial c_\varepsilon} = c^{1-\bar{\theta}}\, c_\varepsilon^{\bar{\theta}-1} - \hbar\, p_\varepsilon = 0, \quad \varepsilon = n+1, \dots, \quad \bar{n},$$

$$\frac{\partial \mathcal{L}}{\partial \hbar} = \chi\, \hat{y} - \sum_{\varepsilon=n+1}^{\bar{n}} p_\varepsilon\, c_\varepsilon = 0. \quad (20)$$

From the first equations we have:

$$c^{1-\bar{\theta}}\, c_\varepsilon^{\bar{\theta}} - \hbar\, p_\varepsilon\, c_\varepsilon = 0.$$

From the above equations and the last one in (20), we solve:

$$\hbar = \frac{c}{\chi \, \hat{y}}.$$

Hence, we have:

$$c^{1-\bar{\theta}} \, c_\varepsilon^{\bar{\theta}-1} - \frac{p_\varepsilon}{\chi \, \hat{y}} = 0 \, , \ \varepsilon = n+1 \, , \dots, \ \bar{n} \ . \ (21)$$

From (21), we have:

$$\frac{c_\varepsilon}{c_\gamma} = \left(\frac{p_\varepsilon}{p_\gamma}\right)^{1/(\bar{\theta}-1)} = \left(\frac{p_\varepsilon}{p_\gamma}\right)^{-\omega} , \ \bar{\theta} = 1 - \frac{1}{\omega} , \ \varepsilon, \ \gamma = n+1 \, , \dots, \ \bar{n} \, . \ (22)$$

where ω is the elasticity of substitution between any two varieties:

$$\frac{\left({p_\varepsilon}/{p_\gamma}\right)}{\left({c_\varepsilon}/{c_\gamma}\right)} \frac{d\left({c_\varepsilon}/{c_\gamma}\right)}{d\left({p_\varepsilon}/{p_\gamma}\right)} = \omega \, .$$

From (20) and (22), we have:

$$\chi \, \hat{y} = \sum_{\varepsilon=n+1}^{\bar{n}} p_\varepsilon \, c_\varepsilon = \frac{c_\gamma}{p_\gamma^{1/(\bar{\theta}-1)}} \sum_{\varepsilon=n+1}^{\bar{n}} p_\varepsilon^{\bar{\theta}/(\bar{\theta}-1)} \quad .$$

Hence, we have:

$$c_\varepsilon = \chi \, \hat{y} \, p_\varepsilon^{1/(\bar{\theta}-1)} \left(\sum_{\substack{\varepsilon \\ m=n+1}}^{\bar{n}} p_m^{\bar{\theta}/(\bar{\theta}-1)} \right)^{-1} . \ (23)$$

From (23) and (19), We have:

$$p = \frac{1}{c} \sum_{\varepsilon=n+1}^{\bar{n}} p_\varepsilon \, c_\varepsilon = \left(\sum_{\varepsilon=n+1}^{\bar{n}} c_\varepsilon^{\bar{\theta}}(t) \right)^{-\frac{1}{\bar{\theta}}} \sum_{\varepsilon=n+1}^{\bar{n}} p_\varepsilon \, c_\varepsilon = \left(\sum_{\varepsilon=n+1}^{\bar{n}} p_\varepsilon^{\bar{\theta}/(\bar{\theta}-1)} \right)^{1-\frac{1}{\bar{\theta}}} .$$

The change in the household's wealth saving minus dissaving:

$$\dot{k}(t) = s(t) - \bar{k}(t). \quad (24)$$

The share of variety ε in the total value of intermediates for consumption is:

$$\varphi_\varepsilon(t) \equiv \frac{x_\varepsilon(t) \, p_\varepsilon(t)}{\sum_{m=n+1}^{\bar{n}} c_m(t) \, p_m(t)}, \quad \varepsilon = n+1, \dots, \bar{n} \, . \quad (25)$$

Insert (23) in (25):

$$\varphi_\varepsilon = p_\varepsilon^{\bar{\theta}/(\bar{\theta}-1)} \left(\sum_{m=n+1}^{\bar{n}} p_m^{\bar{\theta}/(\bar{\theta}-1)} \right)^{-1} . \quad (26)$$

The middle goods sector for consumption

Similar for the middle goods sector for the final good sector, we assume that the production of the middle goods sector for consumers is oligopolistic price competition. The producer of variety ε has the following profit:

$$\pi_\varepsilon(t) = [p_\varepsilon(t) - a_c w(t)] \frac{\varphi_\varepsilon(t)\, \chi\, \hat{y}(t)\, N}{p_\varepsilon(t)}, \quad \varepsilon = n+1, \dots, \bar{n}, \quad (27)$$

where a_c is the unit labor requirement for production of intermediates. Insert (26) in (27):

$$\pi_\varepsilon(t) = [p_\varepsilon(t) - a_c w(t)]\, \chi\, \hat{y}(t)\, N\, p_\varepsilon^{1/(\bar{\theta}-1)}(t) \left(\sum_{m=n+1}^{\bar{n}} p_m^{\bar{\theta}/(\bar{\theta}-1)}(t) \right)^{-1} . \quad (28)$$

The first-order condition (i.e., $\partial\pi_\varepsilon/\partial p_\varepsilon = 0$) results in the following fixed-markup pricing rule:

$$\bar{\theta}\, p_\varepsilon(t) = a_c w(t) . \quad (29)$$

It should be noted that we obtain (29) under the consideration that the number of varieties for consumption, $\bar{n} - n$, is so large that a firm's action has negligible effect on p, or $\sum_{m=n+1}^{\bar{n}} p_m^{\bar{\theta}/(\bar{\theta}-1)}$. From (29) we see that prices are independent of varieties, we call it $p_c(t)$:

$$p_c(t) = \frac{a_c w(t)}{\bar{\theta}} . \quad (29)$$

By (23) the household consumes each variety as follows:

$$c_c(t) = \frac{\chi\, \hat{y}(t)}{(\bar{n} - n)\, p_c\ (t)} . \quad (30)$$

From (28) and (29), we express the profit per firm as follows:

$$\pi_c(t) = \frac{(1 - \bar{\theta})\, \chi\, \hat{y}(t)\, N}{\bar{n} - n} . \quad (31)$$

The total profit of the sector for consumption is:

$$\bar{\pi}_c(t) = (\bar{n} - n)\, \pi_c(t). \quad (32)$$

The profit each household receives is:

$$\pi(t) = \frac{\bar{\pi}_c(t) + \bar{\pi}_i(t)}{N} . \quad (33)$$

Demand and supply of final good

As change in capital stock is equal to the output of the final good sector minus the depreciation of capital stock and total consumption of capital good, we have:

$$\dot{K}(t) = F_i(t) - C_i(t) - \delta_k K(t), \quad (34)$$

where $C_i(t) = c_i(t)N$.

Labor and capital are fully employed

The labor force of intermediate goods sector is:

$$N_x(t) = N_I(t) + N_c(t), \, (35)$$

where $N_I(t)$ and $N_c(t)$ are the labor force employed by the intermediates sectors, respectively, for final good sector and for consumption:

$$N_I(t) = a_N \, x(t) \, n, \;\; N_c(t) = a_c \, c_c(t) \, N \, (\bar{n} - n) \, .$$

The labor force is fully employed:

$$N_i(t) + N_x(t) = hN, \quad (36)$$

National capital equaling national wealth

The value of physical capital is equal to the value of national wealth:

$$\bar{k}(t)N = K(t). \quad (37)$$

We built the model. The model is based on the Solow model, the modeling framework by Dixit-Stiglitz model, and the Grossman-Helpman model with Zhang's concept of disposable income and utility function. We now study properties of the model.

3. THE DYNAMIC PROPERTIES OF THE MODEL

The previous section built a neoclassical growth model by integrating the Uzawa two sector growth model and the Dixit-Stiglitz model with monopolistic competition. The economy is characterized by the neoclassical growth mechanism with perfect competition and monopolistic competition. The following lemma gives a computational program for following the movement of the economic system.

Lemma

The following differential equation determines the motion of the economic system:

$$\dot{x}(t) = \left(\frac{dv\big(x(t)\big)}{dx}\right)^{-1} f\big(x(t)\big), \quad (38)$$

where functions $v\big(x(t)\big)$ and $f\big(x(t)\big)$ are defined in the Appendix. We determine all the other variables as functions of $x(t)$ as follows: $\bar{k}(t)$ with (A14) $\rightarrow$ $K(t) = \bar{k}(t)N$ $\rightarrow$ $w(t)$ by (A7) $\rightarrow$ $r(t)$ by (A8) $\rightarrow$ $X_i(t)$ by (14) $\rightarrow$ $F_i(t)$ by (A6) $\rightarrow$ $N_i(t)$ by (4) $\rightarrow$ $\hat{y}(t)$ by (A11) $\rightarrow$ $p_\varepsilon(t)$ and $p_c(t)$ by (A2) $\rightarrow$ $c_i(t)$, $c(t)$ and $s(t)$ by (18) $\rightarrow$ $c_c(t)$ by (30) $\rightarrow$ $N_i(t)$ and $N_c(t)$ by (32) $\rightarrow$ $\varphi(t)$ by (26) $\rightarrow$ $\pi_i(t)$ by (14) $\rightarrow$ $\bar{\pi}_i(t)$ by (15) $\rightarrow$ $\pi_c(t)$ by (14) $\rightarrow$ $\bar{\pi}_c(t)$ by (15).

As the expressions are complicated, we show dynamic behavior of the system by simulation. We specify the parameters as follows:

$$n = 100,\ \bar{n} = 300,\ N = 10,\ \theta = 0.7,\ \bar{\theta} = 0.8,\ a_N = 0.1,\ a_c = 0.1,\ A_i = 1.2,\ \alpha_i = 0.3,\ \beta_i = 0.4,\ \lambda_0 = 0.7,\ \xi_0 = 0.2,\ \chi_0 = 0.1,\ h = 1.5,\ \delta_k = 0.01\,. \quad (29)$$

The population is 10. The level of human capital is 1.5. The number of varieties of intermediate inputs for final good is 100. The number of varieties of intermediate inputs for consumption is 300. The initial condition is as follows: $x(0) = 0.26$. The simulation result is plotted in Figure 1. From the initial state, the output level of the final good sector and national wealth fall. The two middle goods sectors have lower profits over time. The aggregate input of intermediates rises and its price falls. Each firm in the middle goods sector for final good is expanded; while Each firm in the middle goods sector for consumption is shrunk. The rate of interest rises, while the wage rate falls. Each household gets less profit share. The labor force used by the middle goods sector falls, while the other two sectors employ more labor force. The household has less wealth and consume less final good and intermediates. The utility level falls.

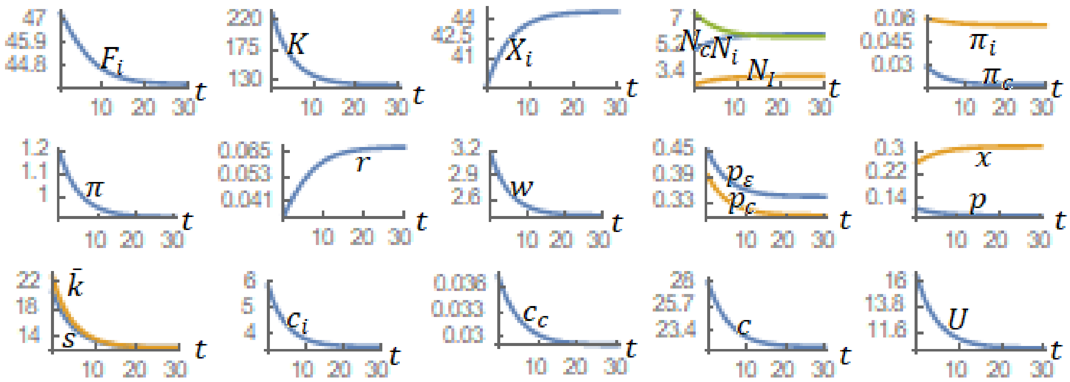

Figure 1. The Motion of the Economic System

The simulation shows that the system becomes stationary in the long term. The simulation confirms the equilibrium point as follows:

$$F_i = 43.94,\ K = 123.1,\ X_i = 44.7,\ N_i = 6.02,\ N_I = 3.18,\ N_c = 5.82,\ x = 0.32,$$
$$\pi_i = 0.057,\ \pi_c = 0.018,\ \pi = 0.92,\ r = 0.067,\ w = 2.42,\ p_\varepsilon = 0.35,\ p_c = 0.3,$$
$$p = 0.08,\ s = \bar{k} = 12.31,\ c_i = 3.52,\ c_c = 0.029,\ c = 21.88,\ U = 10.2.$$

The eigenvalue at the equilibrium point is -0.194. This implies that the equilibrium point is locally stable. We can thus effectively carry out dynamic comparative analysis.

4. COMPARATIVE DYNAMIC ANALYSIS

The previous section showed the movement of the national economy. It is important to follow dynamic effects on the economic system when some exogenous conditions such as resources, technologies, and preferences are shifted. It is straightforward to carry out comparative dynamic analysis as the Lemma gives a computational procedure to calibrate the model and the system has a unique stable equilibrium point. We now study effects of changes in any parameter on transitory processes and equilibrium values of the dynamics. We introduce a symbol $\bar{\Delta}x_j(t)$ to stand for the change rate of the variable, $x_j(t)$, in percentage due to changes in the parameter value.

4.1. A RISE IN DEGREE OF SPECIALIZATION OF THE MIDDLE GOODS SECTOR FOR FINAL GOOD

We first examine the impact of the following change in the degree of specialization of the middle goods sector for final good on the economic system: $n: 100 \Rightarrow 105$. The simulation result is plotted in Figure 2. The output level of the final good sector and total capital fall initially and rise slightly in the long term. The final good and middle goods sectors initially employ more labor and the middle goods sector for consumption initially employs less labor force. The long term the labor force distribution is slightly affected. The aggregated input of intermediates to the final good sector rises but each firm's output falls. The prices of the intermediates are reduced initially and increased in the long term. The rate of interest is enhanced initially and is not affected in the long term. The wage rate is reduced initially and enhanced in the long term. The household's wealth is reduced initially and is augmented slightly. The household consumes less final good and each intermediate capital good initially and almost the same amount in the long term. The utility level falls initially and rises slightly in the long term. The aggregated input of intermediate goods for final good is expanded. The aggregated input of intermediate goods for consumption is reduced. It should be noted that this occurs as $\bar{n}$ is reduced. The output of each variety for final good falls. The output of each variety for consumption is initially reduced and increased in the long term. The profit of each middle goods sector for final good falls. The profit of each middle goods sector for consumption rises.

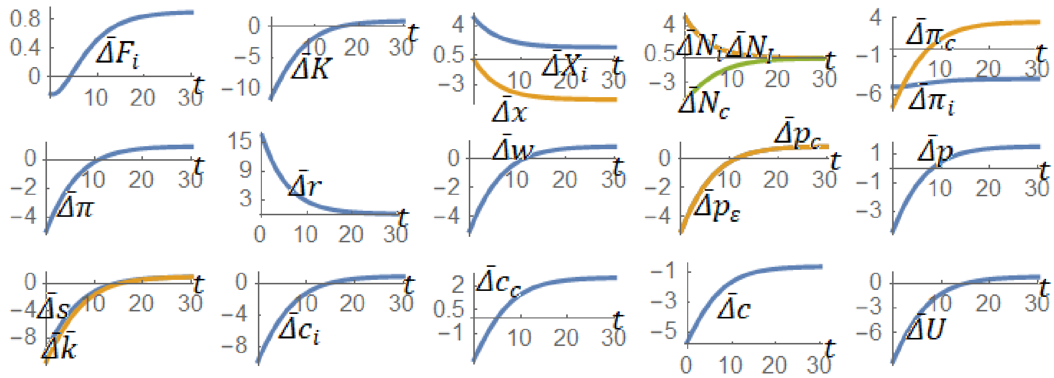

Figure 2. A Rise in Degree of Specialization of the Middle Goods Sector for Final Good

We also change the degree of specialization of the middle goods sector for consumption as follows: $\bar{n}$ = 300 to 310. The change causes constant proportional falls in c_c, p_c and π_c, and proportional rises in U and c. The other variables are not affected.

4.2. THE OUT

We now analyze what happens to the economic system when the output elasticity of intermediate inputs for final good is enhanced as follows:: $\theta: 0.7 \Rightarrow 0.72$. The simulation result is plotted in Figure 3. The output level of the final good sector falls. The total capital rises initially and soon falls. The final good sector employs less labor force. The middle goods sector for final good employs more labor. The middle goods sector for consumption initially employs more labor force and late on less. The aggregated input of intermediates to the final good sector falls. Each firm in the middle goods sector for final good produces more. The aggregated consumption of intermediates is augmented initially and is reduced late on. Each firm in the middle goods sector for consumption produces more initially and less late on. The prices of the intermediates are reduced. The rate of interest is reduced initially and is augmented in the long term. The wage rate is reduced. The household's wealth is augmented slightly initially and is reduced late on. The household consumes less final good. The utility level is reduced. The profit of every middle goods firm is reduced.

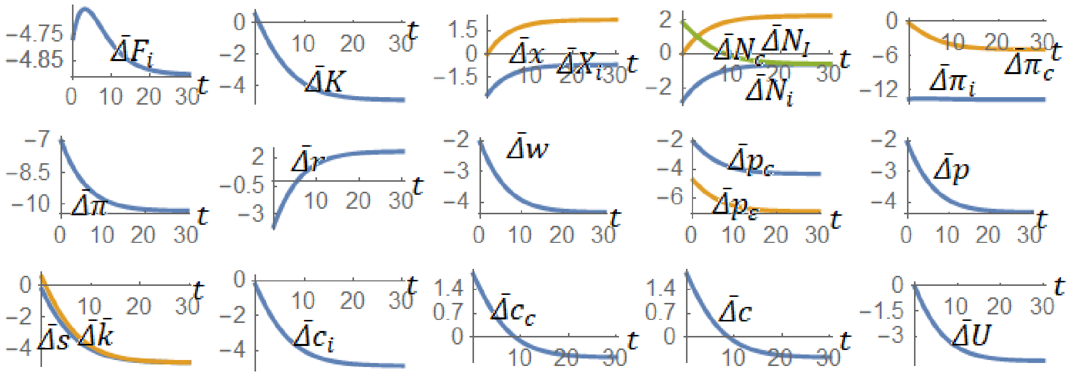

Figure 3. The Output Elasticity of Intermediate Inputs for Final Good Is Increased

4.3. THE UTILITY ELASTICITY OF INTERMEDIATES FOR CONSUMPTION IS AUGMENTED

We now analyze what happens to the economic system when the output elasticity of intermediate inputs for consumption is increased as follows: $\bar{\theta}$: $0.8 \Rightarrow 0.81$. The simulation result is plotted in Figure 4. The output level of the final good sector falls. The total capital decreases. The final good sector and middle good sector for final good employ less labor force. The middle goods sector for consumption employs more labor force. The aggregated input of intermediates to the final good sector falls. Each firm in the middle goods sector for final good produces less. The aggregated consumption of intermediates is reduced. Each firm in the middle goods sector for consumption produces more. The price of intermediates for final good rises. The rate of interest is increased initially and is changed slightly in the long term. The wage rate is reduced initially and enhanced slightly late on. The household's wealth is reduced. The household consumes less final good. The utility level is reduced. The profit of each middle goods firm is reduced.

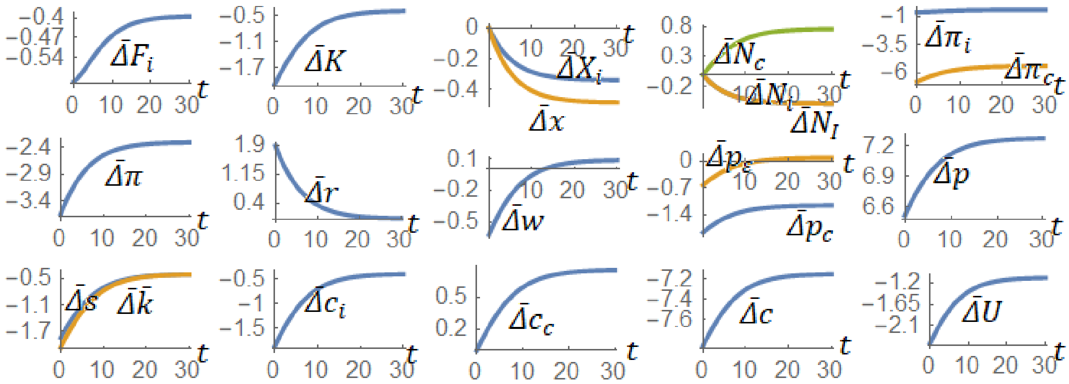

Figure 4. The Utility Elasticity of Intermediates for Consumption Is Increased

4.4. THE UNIT LABOR OF THE MIDDLE GOODS SECTOR FOR FINAL GOOD IS INCREASED

We now study the impact that the unit labor of the middle goods sector for final good is increased as follows: a_N: $0.1 \Rightarrow 0.11$. The simulation result is plotted in Figure 5. The output level of the final good sector falls. The total capital decreases. The final good sector and middle good sector for final good employ less labor force. The middle goods sector for consumption employs more labor force. The aggregated input of intermediates to the final good sector falls. Each firm in the middle goods sector for final good produces less. The aggregated consumption of intermediates is reduced. Each firm in the middle goods sector for consumption produces more. The price of intermediates for final good rises. The rate of interest is increased initially and is changed slightly in the long term. The wage rate is reduced initially and enhanced slightly late on. The household's wealth is reduced. The household consumes less final good. The utility level is reduced. The profit of each middle goods firm is reduced.

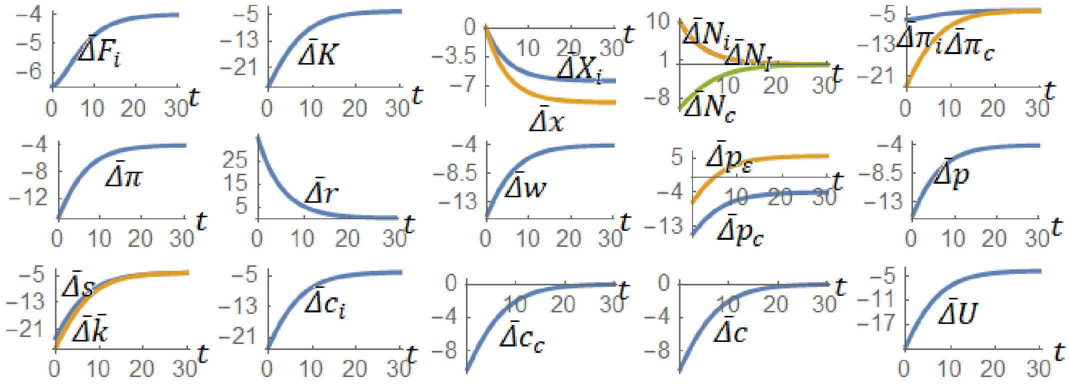

Figure 5. The Unit Labor of the Middle Goods Sector for Final Good is Increased

4.5 THE PROPENSITY TO CONSUME MIDDLE GOODS FOR CONSUMPTION IS INCREASED

We now examine the impact that the following rise in the propensity to consume middle goods for consumption: $\chi_0 = 0.1 \Rightarrow 0.11$. The simulation result is plotted in Figure 6. The national wealth and output level of the final good sector are reduced. The final good sector and the middle goods sector for consumption employ more labor force. The middle goods sectors for final good employs less labor force. The aggregated input of intermediates to the final good sector falls. Each firm in the middle goods sector for final good produces less. The aggregated consumption of intermediates is augmented. Each firm in the middle goods sector for consumption produces more. The prices of intermediates fall initially and rise late on. The rate of interest is increased initially and is changed slightly in the long term. The wage rate is reduced initially and enhanced slightly in the long term. The household's wealth is reduced. The household consumes less final good. The utility level is reduced initially and is slightly increased late on. The profit of each middle goods firm for consumption falls initially and rise in the long term. The profit of each middle goods firm for final goods falls.

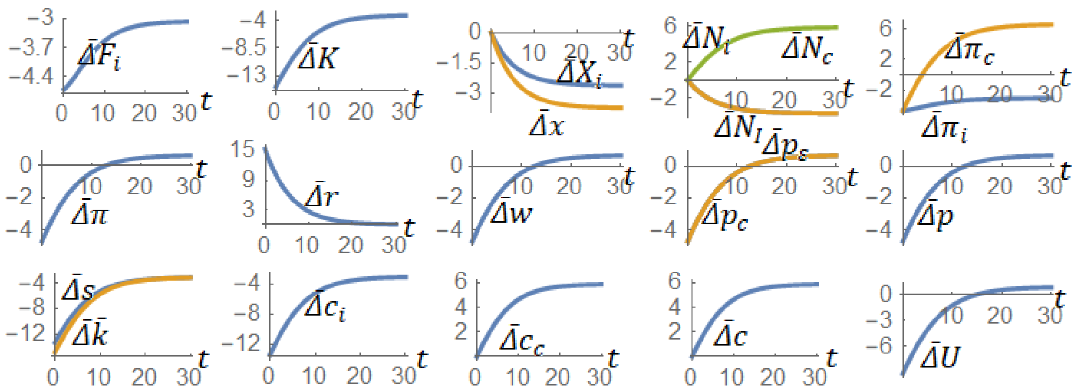

Figure 6. The Propensity to Consume Middle Goods for Consumption is Increased

4.6. THE PROPENSITY TO SAVE IS ENHANCED

We now study what will happen to the economic system if the propensity to save is increased as follows: $\lambda_0 = 0.7 \Rightarrow 0.71$. The simulation result is plotted in Figure 7. The national wealth and output level of the final good sector are augmented. The final good sector and middle good sector for final good employ less labor force initially and more in the long term. The middle goods sector for consumption employs more labor force. The aggregated input of intermediates to the final good sector falls initially and rises slightly in the long term. Each firm in the middle goods sector for final good produces less initially and slightly more in the long term. The aggregated consumption of intermediates is reduced, and its price is increased. Each firm in the middle goods sector for consumption produces less initially and more in the long term. The prices of intermediates are enhanced. The rate of interest falls. The wage rate is enhanced. The household's wealth is increased. The household consumes more final good. The utility level is increased. The profit of each middle goods firm is increased.

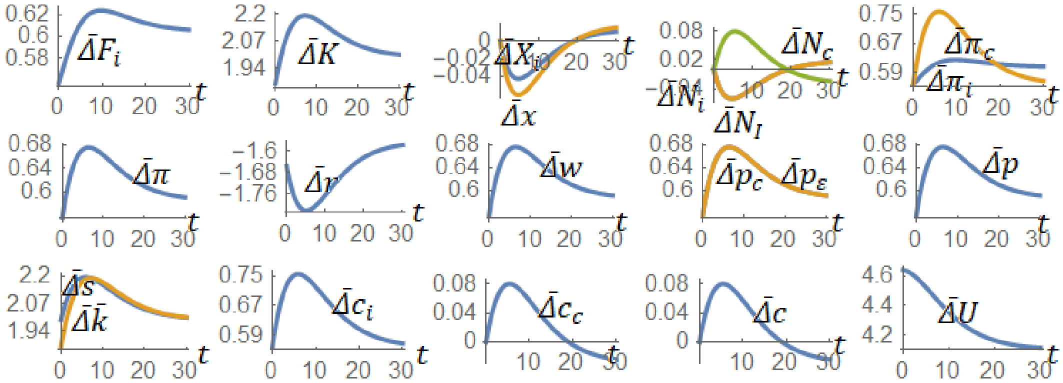

Figure 7. The Propensity to Save Is Increased

4.7. THE HUMAN CAPITAL IS AUGMENTED

We now deal with the impact that the following rise in human capital has on the economic system: $h = 1.5 \Rightarrow 1.6$. The simulation result is plotted in Figure 8. The output level of the final good sector and national wealth are enhanced. All the sectors and all the firms employ more labor force. The aggregated input of intermediates to the final good sector rises. Each firm in the middle goods sector for final good produces more. The aggregated consumption of intermediates is augmented. Each firm in the middle goods sector for consumption produces more. The prices of intermediates are increased initially and reduced in the long term. The rate of interest falls initially and does not change in the long term. The wage rate rises initially and falls late on. The household's wealth is increased. The household consumes more final good. The utility level is enhanced. The profit of each middle goods firm is increased.

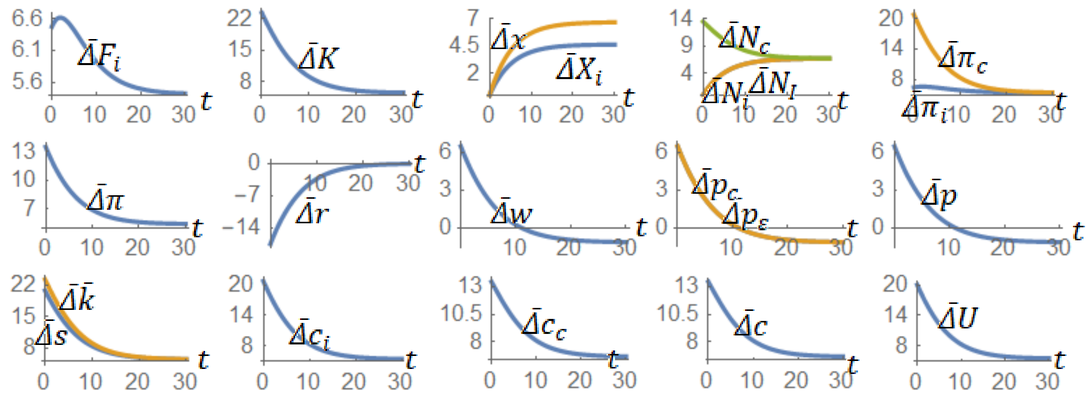

Figure 8. The Human Capital Is Augmented

5. CONCLUDING REMARKS

This study integrated the three basic models in neoclassical growth theory with perfect competition and general equilibrium theory with monopolistic competition. The most important model in neoclassical growth theory is the Solow one-sector growth model (1956). The two important model in contemporary general equilibrium theory with imperfect competition are respectively by Dixit and Stiglitz (1977) and Grossman and Helpman (1990). The economy in our approach is composed of three sectors – the final good sector as Solow's one sector, the middle goods sector for final good as in Grossman and Helpman's intermediates sector (which supplies intermediate inputs for the final good sector) , and the middle goods sector for consumption as in Dixit and Stiglitz's intermediates sector (which supplies goods for consumption). Our model is different from the Solow model in that we model the household behavior with the utility function and disposable income proposed by Zhang. We deviate from the Dixit-Stiglitz and the Grossman- Helpman model in that we distribute the profits of firms in monopolistic competition to households. Our distribution of profits is just another possible distribution of profit sharing. Profits may be shared by different agents for different uses. Our economy is characterized by co-existence of perfect and monopolistic competition. We contribute to the literature by integrating the two main modelling frameworks in economic theory. We built and then simulated the model. We demonstrated a unique stable equilibrium point. We also plotted the motion of the economy. We examined the effects of changes in, respectively, degree of specialization of the middle goods sector for final good, the output elasticity of intermediate inputs for final good, the utility elasticity of intermediates for consumption, the unit labor of the middle goods sector for final good, the propensity to consume middle goods for consumption, the propensity to save, and the human capital. The comparative dynamic analysis analyzes the effects of the exogenous changes on transitory process and long-term equilibrium structure. As there are a large amount of publications in the two mainstreams and our model is developed on the basic models in the literature, we can extend and generalize the model by introducing more refined or special features in the literature. For instance, we may make human capital endogenous variables as in the Rome model. We may also invest the profits of the intermediates sectors in R&D activities as in the Grossman- Helpman model.

Appendix: Proving Lemma

We now confirm the Lemma. We introduce a variable

$$z \equiv \frac{r + \delta_k}{w}.$$

From (3) we get

$$z \equiv \frac{r + \delta_k}{w} = \frac{\bar{\beta}_i N_i}{K}, \quad (A1)$$

where $\bar{\beta}_i \equiv \alpha_i / \beta_i$. From (12) and (29), we have

$$p_\varepsilon = \frac{a_N w}{\theta}, \quad p_c = \frac{a_c w}{\bar{\theta}}. \quad (A2)$$

From (3) and (14), we solve

$$w = \frac{\beta_i}{N_i} \frac{p_\varepsilon n x}{\gamma_i \theta}, \quad (A3)$$

where we also use (14). With (A2) and (A3), we get

$$N_i = a\,x, \quad (A4)$$

where

$$a = \frac{\beta_i\, n\, a_N}{\gamma_i\, \theta^2}.$$

By (2) we have

$$F_i = A_i N_i \left(\frac{K_i}{N_i}\right)^{\alpha_i} \left(\frac{X_i}{N_i}\right)^{\gamma_i}. \quad (A5)$$

Insert (A1) and (A4) in (A5)

$$F_i(x,z) = A_i\, a\, x \left(\frac{\bar{\beta}_i}{z}\right)^{\alpha_i} \left(\frac{n x^{\theta-1}}{a}\right)^{\gamma_i}. \quad (A6)$$

From (3), (A8) and (A4), we have

$$w(x,z) = \frac{\beta_i F_i(x,z)}{ax} = \bar{A}_i \left(\frac{1}{z}\right)^{\alpha_i} x^{(\theta-1)\,\gamma_i}, \quad (A7)$$

where we use (A6) and

$$\bar{A}_i \equiv A_i\, \beta_i\, \bar{\beta}_i^{\alpha_i} \left(\frac{n}{a}\right)^{\gamma_i}.$$

From (A1) we have:

$$r(x,z) = w\, z - \delta_k. \quad (A8)$$

Insert (A4) in (35):

$$h\, N = (a + a_N\, n)\, x + \frac{\chi\, \bar{\theta}\, N\, \hat{y}}{w}, \quad (A9)$$

in which we also use (30), (35) and (A2). From (33)-(31), (15) and (13), we have:

$$\pi = \frac{(1-\bar{\theta})\,\chi\, \hat{y}\, N + (1-\theta)\,\gamma_i\, F_i}{N}. \quad (A10)$$

Insert (A10) in (14):

$$\hat{y} = (\delta + w\, z)\,\tilde{\chi}\, \bar{k} + \left\{h + \frac{(1-\theta)\,\gamma_i\, a\, x}{\beta_i\, N}\right\} \tilde{\chi}\, w, \quad (A11)$$

where we use $(A7)$ and (A8)

$$\tilde{\chi} \equiv \frac{1}{1 - (1 - \bar{\theta})\chi}, \qquad \delta \equiv 1 - \delta_k.$$

Insert (A11) in (A9):

$$h_0 - h_1 x = \left(\frac{\delta}{w} + z\right) \tilde{\chi} \bar{k}, \quad (A12)$$

where

$$h_0 \equiv \frac{h}{\chi \bar{\theta}} - \tilde{\chi} h, \;\; h_1 \equiv \frac{a + a_N n}{\chi \bar{\theta} N} + \frac{(1-\theta)\gamma_i a \tilde{\chi}}{\beta_i N}.$$

From (37), (34) and (24), we have:

$$(\lambda + \xi)\hat{y} = \frac{a x w}{N \beta_i} + \delta \bar{k}, \quad (A13)$$

where we use (18) and (A7). Insert (A11) in (A13):

$$\bar{k} = (h \tilde{\chi} + \tilde{\chi}_1 x)\{\delta_1 - (\delta + w z)\tilde{\chi}\}^{-1} w, \quad (A14)$$

where

$$\tilde{\chi}_1 \equiv \frac{(1-\theta)\gamma_i \tilde{\chi} a}{\beta_i N} - \frac{a}{(\lambda + \xi) N \beta_i}, \quad \delta_1 = \frac{\delta}{\lambda + \xi}.$$

Insert (A12) in (A14):

$$h_0 - h_1 x = (\delta + z w)\tilde{\chi}(h \tilde{\chi} + \tilde{\chi}_1 x)\{\delta_1 - (\delta + w z)\tilde{\chi}\}^{-1}. \quad (A15)$$

From (A15), we solve:

$$z w = \frac{\delta_1 (h_0 - h_1 x)}{\tilde{\chi}(h \tilde{\chi} + \tilde{\chi}_1 x + h_0 - h_1 x)} - \delta. \quad (A16)$$

Insert (A7) in (A16):

$$z = \varphi(x) \equiv \left\{\left(\frac{\delta_1 (h_0 - h_1 x)}{\tilde{\chi}(h \tilde{\chi} + \tilde{\chi}_1 x + h_0 - h_1 x)} - \delta\right) \frac{1}{\bar{A}_i x^{(\theta-1)\gamma_i}}\right\}^{1/(1-\alpha_i)}. \quad (A17)$$

It is straightforward to confirm that all the variables can be expressed as functions of x by the following procedure: z by (A17) → $\bar{k}$ with (A14) → $K = \bar{k}N$ → w by (A7) → r by (A8) → X_i by (14) → F_i by (A6) → N_i by (4) → $\hat{y}$ by (A11) → p_ε and p_c by (A2) → c_i, c and s by (18) → c_c by (30) → N_i and N_c by (32) → φ by (26) → π_i by (14) → $\bar{\pi}_i$ by (15) → π_c by (14) → $\bar{\pi}_c$ by (15). From this procedure and (22), (A11) and (A5), we have:

$$\dot{\bar{k}} = f(x) \equiv s - \bar{k}. \quad (A18)$$

Denote (A12) by $\bar{k} = v(x)$. We have:

$$\dot{\bar{k}} = \frac{dv}{dx}\dot{x}. \quad (A19)$$

From (A18) and (A19), we have:

$$\dot{x} = \left(\frac{dv}{dx}\right)^{-1} f. \quad (A20)$$

In summary, we proved the Lemma.

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