THE USV ANNALS OF ECONOMICS AND PUBLIC ADMINISTRATION | VOLUME 19, ISSUE 2(30), 2019

STOCHASTIC MODELING OF THE OPTIMAL CREDIT STRATEGY FOR THE ENTERPRISE WITH INCOME LIMITATIONS

Associate Professor PhD **Olena YAROSHENKO**
Faculty of Economics, Yuriy Fedkovych Chernivtsi National University, Ukraine
o.yaroshenko@chnu.edu.ua

Assistant Professor PhD **Taras LUKASHIV**
Faculty of Mathematics and Informatics, Yuriy Fedkovych Chernivtsi National University, Ukraine
t.lukashiv@gmail.com

Abstract:

In a crisis state of the economy, there is an obvious trend towards reduction of activities of small and midsize businesses mainly because of the lack of financial resources for their development. Therefore, the search for the effective and precise mechanisms for management of small and midsize businesses is of great current interest nowadays.

We believe that systematic analysis of principles and methods of economic and mathematical modeling, considering their huge potential, will enable us to understand the reasons of the stated problems better and to examine the performance of the economic system in general as well as the development of complicated processes occurring in it.

The purpose of the paper is to present a stochastic model of optimal credit strategy for an enterprise considering credit payments and income limitations. The model allows identifying the optimal loan amount aimed at maximizing the average income. The structure of the optimal process for the given model is also described.

The methods discussed in the study are appropriate to any decision-making context; still, the principal focus of the research is upon their application in business.

Key words: small and midsize business, economic and mathematical modeling, optimal control, stochastic model, Markov parameter, small and midsize business.

JEL classification: C61, P42

1. INTRODUCTION

Small and midsize businesses (SMBs) constitute an integral part of the market economy of any country and, depending on the degree of their development, allow mobilizing financial and production resources, accelerate the pace of scientific and technical progress, solve the problem of employment and create economically independent and stable middle class.

The need for the development of SMBs is particularly acute in the period of recession when large enterprises cannot adapt quickly to changing conditions and either reduce their turnover or stop production. However, the development of small and midsize businesses under these conditions also slows down without the proper state regulation, due to the poor access to financial and credit resources and some other internal and external problems.

A number of problems hinder the access of SMBs to financial credit resources, namely a high credit cost, a short loan period, the need to have assets to secure the loan, etc. In fact, all of these force SMBs to invest only their own earnings and/or investments of business founders and to seek other ways of managing their economic activities, allowing them to preserve and strengthen their position in the market. Therefore, the investigation of the problem of increasing the efficiency of lending from the point of view of small and midsize businesses and determining the optimal amount of monetary credit, necessary to maximize their average income indisputably is of great scientific and practical interest.

The relevance of the problems of effective functioning of SMBs as well as mechanisms for their credit and financial support are analyzed in a significant number of studies (Yoshino, N. and F. Taghizadeh-Hesary, 2016; Moro, A.; Lucas, M.; Grimm, U. and Grassi, E., 2010). Nonetheless, despite the thorough tackling of the problem in the existing studies, a number of issues remain unresolved or need clarification. Firstly, it concerns building of economic and mathematical models, especially stochastic ones, which take into account the stochastic nature, nonlinearity and dynamism of the development of business activity in the market conditions of management, allow analyzing the activity of enterprises, forecasting their development and making effective management decisions.

Boychuk M. and Yaroshenko O. (2015) built and examined a stochastic model of the optimal behavior of the distributor in the pharmaceutical market as well as described the structure of the optimal process. Relying on that study, in this research we will build a stochastic mathematical model of the optimal strategy for the trading enterprise and define the optimum size of monetary credit, which should be involved in order to maximize the average income of the enterprise in view of various constraints arising from the specific business activity.

2. MODEL BUILDING

First, we will formalize the deterministic model of the optimal credit strategy for the trading enterprise.

Let the enterprise has an opportunity to take a money loan totaling $k(t)$ at a constant interest rate r at any point in time $t \in [0, T]$ (T – planning time-frame). It uses the credit to purchase some amount of goods $v(t)$ (in monetary terms) with the aim of selling them and earning the good profit.

Suppose that the quantity of goods sold at time t is in direct proportion to the volume of goods $v(t)$ with the constant coefficient α and the goods quantity at the time-point $t + \Delta t$ ($\Delta t > 0$ – random delta time) equals the difference, increased by the amount of credit, between the volume of goods at time t and the quantity of goods sales:

$$v(t + \Delta t) = v(t) - \alpha v(t) + k(t).$$

Then we get the balance equation of the dynamics of the available output in the value form:

$$\dot{v}(t) = k(t) - \alpha v(t), t \in [0, T] \qquad (1)$$

This differential equation must be supplemented by the initial condition

$$v(0) = v_0,$$

which means, that at the initial time $t = 0$ the enterprise has the initial stock of goods v_0 items.

Since the credit amount is always a finite quantity, the above ratios should be complemented by the restriction

$$0 \le k(t) \le k_0, \ t \in [0, T],$$

where k_0 – the set upper bound of limitations on the credit amount.

Restrictions are also imposed on the profit on sales of goods

$$\gamma\alpha v(t) - rk(t) \ge \varepsilon, \ t \in [0, T],$$

where γ – gross income margin from the each sold item of goods, $\varepsilon \ge 0$ – minimum profit.

Then the goal of the enterprise is to maximize the accumulated (integral) profit

$$\Phi = \int_0^T [\gamma\alpha v(t) - rk(t)]dt \to \max_{0 \le k \le k_0}$$

under the following conditions.

3. MODEL STUDY

It is clear that the objective function is equivalent to

$$Q = \int_0^T [rk^2(t) - \gamma\alpha v^2(t)]dt \to \min_{0\le k\le k_0} \tag{2}$$

therefore we shall use it in the following calculations.

Let the gain in the goods production $\dot{v}$ be monitored. Obviously, the accuracy of measurements will contain a certain error, and the deviation of the empirical data from their theoretical values is of random character the type of “white noise”, that does not depend on the prehistory and is formalized according to the model (1) when the composed type differential in the Wiener process is introduced. In addition, we will assume that at the moments of time $t \in S = \{t\_k\uparrow, \quad k \in \overline{(0,n)}; t\_0 = 0, t\_n < T$ instantaneous changes of the value v are possible (receipt of new goods, disposition of the expired etc.) with a certain probability. This situation is formalized through the introduction of the Markov-process parameter and due to the examination of the so-called equations with a random structure, which are studied in detail in various formulations (Kats I. Y., 1998; Lukashiv T.O., Yurchenko I.V. and Yasinskii V.K., 2009; Yasinskii V.K., Lukashiv T.O. and Yasinskaya L.I., 2009)

Therefore, to consider a stochastic model of an optimal credit strategy for an enterprise appears to be well grounded.

Let us look at the random process $v(t), t \in [0,T]$ on the probabilistic basis $(\Omega, \mathfrak{F}, F = [\{\mathfrak{I}\}_t, t \in [0,T], P\}$ given by the stochastic differential equation

$$dv(t) = [k(t) - \alpha v(t)]\xi(t)dt + \sigma\xi(t)dw(t), t \in [0,T]\backslash S \tag{3}$$

with the initial conditions

$$v(0) = v_0 \in \mathbb{R}, \xi(0) = y \in Y. \tag{4}$$

Here $\xi(t)$, $t \in [0,T]$ – the Markov process with the values in space $Y = \{y_1, \dots, y_n\}$; $w(t)$, $t \in [0,T]$ – one-dimensional standard Wiener process, where w and ξ – are $\mathfrak{I}_t$-correlated and independent between each other (Dub D., 1963; Dynkin E., 1969).

A scalar process ξ with a finite number of states will be considered as a homogeneous Markov chain, which means almost piecewise constancy of all ξ realizations with the known parameters $q_{ij}, q_i = \sum_{i\neq j} q_{ij}, \quad i = \overline{1,n}, j = \overline{1,n}$. In this case, conditional probabilities admit decomposition

$$P\left\{\xi(t+\Delta t) = {}^{y_j}\Big/_{(\xi(t) = y_i\}} = \right) q_{ij}(t) + o(t),$$

$$P\left\{\xi(\tau) = {}^{y_j,\quad t<\tau<t+\Delta t}\Big/_{(\xi(t) = y_i\}} = \right) 1 - q_i\Delta t + o(\Delta t).$$

Let us look at the behavior of the trajectories of the system (3), (4). At the interval $t \in [\tau - h, \tau), \tau > h > 0$, where $\xi(t) = y_i \in Y$ we will consider the equation as a stochastic differential equation (3)

$$dv(t) = [k(t) - \alpha v(t)]y_i dt + \sigma y_i dw(t), \tag{5}$$

$$v(t-h) = v, y(t-h) = y_i. \tag{6}$$

Then, if τ – the moment of transition of the value $\xi(\tau - 0) = y_i$ to the value $(\tau) = y_j \neq y_i$, then at the next interval of the constancy of the system structure (Kats I. Y., 1998) $\xi(\tau) = y_j$ it is necessary to solve the stochastic differential equation (4) with y_j instead of y_i. This raises the problem of the choice of the initial condition $v(\tau)$ for the new stochastic differential equation

$$dv(t) = [k(t) - \alpha v(t)]y_i dt + \sigma y_i dw(t), t \in [\tau, \tau + h_1).$$

The choice $v(\tau)$ is completely determined by the real properties of the modeled object (Kats I. Y., 1998).

Consequently, the solution v of equation (3), the Markov chain ξ and the initial condition (4) at each interval of the process constancy ξ are determined by the Markov process $\{v(t), \xi(t)\}$, $t \in [0, T]$ (Dynkin E., 1969; Kats I. Y., 1998), in which the random component $v(t) \in \mathbb{R}, t \in [0, T]$ characterizes changes in the state of the system (volume of goods), and ξ – a stochastic variation in its structure. This explains the definition of the system (3) as a system of random structure (Kats I. Y., 1998).

The following variations of the behavior of the trajectory of the strong solution of the stochastic differential equation (3) with the initial condition $v(0) = v_0$, $\xi(0) = y \in Y$ (Kats I. Y., 1998; Lukashiv T. O., Yurchenko I. V. and Yasinskii V. K., 2009) are of the greatest interest in most cases.

1. At the moment τ of the discontinuous jump in the structure ξ the phase variable v changes continuously with the probability 1:

$$v(\tau - 0) = v(\tau). \tag{7}$$

2. At the moment $\tau > 0$ of the discontinuous jump in the structure ξ the phase variable v is uniquely determined by the state in which the system was just before the structure change and the transition $\xi(\tau - 0) = y_i$ in $(\tau) = y_j \neq y_i$. Here, it is natural to assume that

$$v(\tau) = \varphi_{ij}(v(\tau - 0)), i \neq j. \tag{8}$$

where $\varphi_{ij}(x)$, when such constants exist K_{ij} , that

$$v(\tau) = K_{ij} v(\tau - 0). \tag{0}$$

It is necessary to mention that case 1 (indiscreet with probability 1 of the change of the phase variable) is obtained from (9) for $K_{ij} = 1$.

It should also be noted that in a certain specific case variants 1 or 2 occur depending on the time discretization of the investigation of the posed problem.

The task to choose the optimal trajectory for the enterprise (3), (4), (2) we will do using the Bellman method (Andreeva E. A., Kolmanovskiy E. B. and Shayhet L. E., 1992; Lukashiv T.O., Yasinskaya L.I. and Yasinskii V.K., 2008)

The Bellman equation for system (3) will be

$$\inf_k [LV(t, v) - rk(t)] = 0 \tag{10}$$

with the initial condition

$$V(T, v(T)) = 0, \tag{11}$$

where V – definite at $[0, T] \times \mathbb{R}_+$ continuously differentiated once at t and twice at v function, L – weak infinitesimal operator, determined according to (3) (Kats I. Y., 1998).

$$LV_i(t, v) = \frac{\partial}{\partial t} V(t, v) + [-\alpha y_i + y_i k] \frac{\partial V}{\partial t} + \\ + \frac{1}{2} \frac{\partial^2 V}{\partial t^2} \sigma^2 y_i^2 + \sum_{j \neq i}^{l} [[V]_j(t, v) - V_i(t, v)] q_{ij}, i = \overline{1, n} \tag{12}$$

Here the index near V governs the state of the Markov process ξ .

The functional V_i that satisfies equation (10) for the control problem (3), (2) will be sought in the form

$$V_i(t,v) = v^2(t)P(y_i). \tag{13}$$

Substituting (12) for (10) considering (2), we will find that the optimal control $k_i^0, i=\overline{1,n}$ must have the following form

$$k_i^0(t,v) = -\frac{1}{2}\frac{1}{r}y_i\frac{\partial V_i}{\partial t}, i=\overline{1,n} \tag{14}$$

We substitute (13) for (10). Taking into account (14) and the notation $P_i = P(y_i), i=\overline{1,n}$ eliminating k_i^0 and equating the resulting quadratic form to zero (Andreeva E. A., Kolmanovskiy E. B. and Shayhet L. E., 1992; Lukashiv T.O., Yasinskaya L.I. and Yasinskii V.K., 2008; Yasinskii V.K., Lukashiv T.O. and Yasinskaya L.I., 2009), we obtain a system of equations

$$P_i - 2\alpha y_i P_i - \frac{2y_i^2}{r}P_i^2 + \sigma^2 y_i^2 P_i + \sum_{j=i}^{n}\left[[P]_j K_{ij} - P_i\right]q_{ij} - \gamma\alpha = 0, i=\overline{1,n} \tag{15}$$

and when we decide it we will find the unknown P_i to determine the optimal loan volume.

4. MODEL IMPLEMENTATION

We will exemplify the offered procedure by an example.

Suppose that at the moments of time $t_0, t_1, \dots, t_n$, where $t_0 = 0$ and $t_n = T$ changes in the volume of goods in the warehouse occur in the cash equivalent (shipment or receipt) and the dynamics of the product volume is described by the system (3), (4).

The function $V_i, i=\overline{1,n}$ we will choose in the type

$V_i(v) = c_i x^2, i=\overline{1,n}$

Suppose that ξ has two values, that is $\xi(t) \in \{y_1, y_2\}$. Then the system of quadratic equations (15) has the form

$$c_i - 2\alpha y_i c_i - \frac{2c_i^2}{r}y_i^2 + \sigma^2 y_i^2 c_i + \left[[c]_j - c_i\right]q_{ij} - \gamma\alpha = 0, i \in \{1,2\}, \quad j \in \{1,2\}, \quad i \neq j.$$

By assuming that the ratio of the sold and available goods $\alpha = 0,6$, the loan interest rate $r = 0{,}25$ the margin profit ratio $\gamma = 0,2$, and the parameters $\sigma = 2$, $y_1 = 2$, $y_2 = 2,5$, $q_{12} = q_{21} = 0,5$ then you can find its solution $c_1 = -0{,}55, c_2 = -0,02$.

Then, with the volume of output (in the cash equivalent) $v = 150\,000$ and the desired amount of the loan $k_0 = 50\,000$ the optimal loan amount that must be obtained:

1) for a period $[t_0, t_1]$ it should be calculated in this way

$$k_1^0(t,v) = min\left\{-\frac{1}{r}y_1 c_1 v; k_0\right\} = 50\,000.$$

2) for a period $[t_1, t_2]$ it should be calculated in this way

$$k_2^0(t,v) = min\left\{-\frac{1}{r}y_2 c_2 v; k_0\right\} = 30\,000.$$

5. CONCLUSIONS

Thus, a mathematical model of the optimal credit strategy for a trading enterprise is constructed in this research. It is described by a stochastic differential equation with a Markov parameter, characterizing the change in the volume of goods. The model allows identifying the optimal monetary loan amount that should be obtained and the corresponding optimal quantity of goods that comply with the differential model of the existing quantity of goods and maximize the average integral income. Also this differential model supplemented by the initial condition of prehistory of volume of goods, the limitation on the amount of credit and profit received from the product selling. An algorithm for solving the problem using the Bellman method is constructed and illustrated with a model example.

In fact, that model demonstrates one of the possible methods of modeling the optimal behavior of an enterprise in the market. In our opinion, a systematic approach is required for the effective management in this sector, that is, market research as an aggregate system. This will lead to the construction of a complex of economic and mathematical models of both structural components and the system as a whole. The analysis of such models and arriving at the optimal economic decisions, together with the desired consistent policy of the state, can become a powerful factor in economic reforms, help overcome the decline in production and create the prerequisites for economic recovery.

BIBLIOGRAPHY

1. Andreeva E. A., Kolmanovskiy E. B. and Shayhet L. E. (1992) Management systems with aftereffect, Moscow, Nauka.
2. Boychuk M. and Yaroshenko O. (2015) “Stochastic modeling of optimized credit strategy of a distributing company on the pharmaceutical market“ Bulletin of Taras Shevchenko National University of Kyiv, Economics, 11 (176), 49-54.
3. Dub D. (1963) Veroyatnostnye protsessy [Stochastic processes]. Moscow, Fizmatgiz.
4. Dynkin E. (1969) Markovskie protsessy [Markov Process]. Moscow, Fizmatgiz.
5. Kats I. Y. (1998) “Metod funktsiy Lyapunova v zadachah ustoychivosti i stabilizatsii sistem sluchaynoy struktury” [The method of Lyapunov functions in problems of stability and stabilization of systems of random structure]. Ekaterinburg, Ural University of Railway Transport.
6. Lukashiv T.O., Yasinskaya L.I. and Yasinskii V.K. (2008) “Synthesis of the Optimal Control for Linear Stochastic Dynamical Systems with Finite Aftereffect and Poisson Disturbances”. Journal of Automation and Information Sciences, 10 (450), 22-37.
7. Lukashiv T.O., Yurchenko I.V. and Yasinskii V.K. (2009) “Lyapunov function method for investigation of stability of stochastic Ito random-structure systems with impulse Markov switchings. I. General theorems on the stability of stochastic impulse systems”. Cybernetics and Systems Analysis, 2 (45), 281-290.
8. Moro, A.; Lucas, Michael; Grimm, U. and Grassi, E. (2010). Financing SMEs: a model for optimising the capital structure. In: 17th Annual Global Finance Conference, 27-30 Jun 2010, Poznan
9. Yasinskii V.K., Lukashiv T.O. and Yasinskaya L.I. (2009) “Stabilization of Stochastic Diffusive Dynamical Systems with Impulse Markov Switchings and Parameters. Part II. Stabilization of Dynamical Systems of Random Structure with External Markov Switchings”. Journal of Automation and Information Sciences, 2009, 4 (41), 26-42.
10. Yoshino, N. and F. Taghizadeh-Hesary. 2016. Major Challenges Facing Small and Medium sized Enterprises in Asia and Solutions for Mitigating Them. ADBI Working Paper 564. Tokyo: Asian Development Bank Institute. Available: http://www.adb.org/publications/majorchallenges-facing-small-and-medium-sized-enterprises-asia-and-solutions/